2.001 - MECHANICS AND MATERIALS I
Lecture #27
12/13/2006
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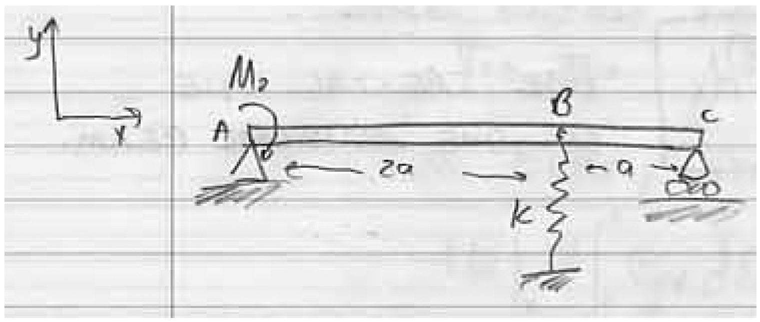
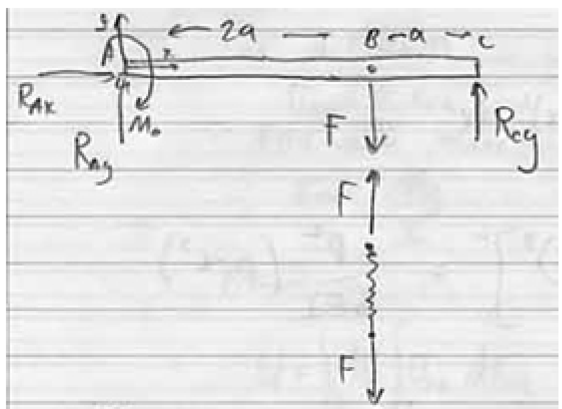
Example Problems



Q: What is the force in the spring?

FBD

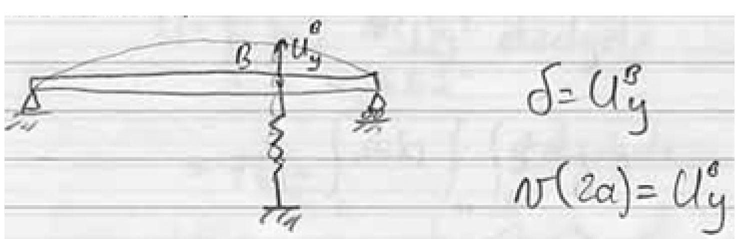


4 unknowns
3 equilibrium equations
⇒ statically indeterminate.
Force-Deformation

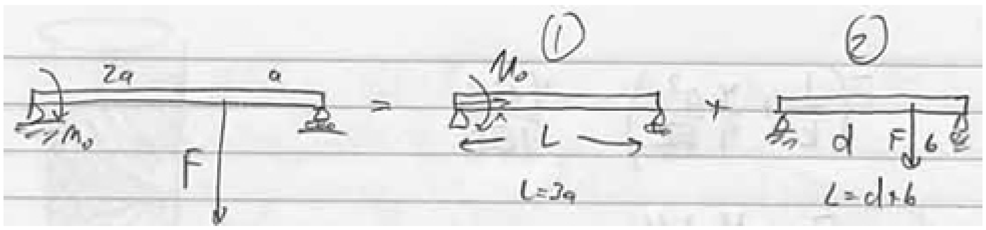
$$F = k\delta$$

$$\frac{d^2v(x)}{dx^2} = \frac{M(x)}{EI}$$

Compatibility:



$$\delta = u_y^B$$
$$v(2a) = u_y^B$$



$$v_M(x) = \frac{-M_D x}{6EIL}(x^2 - 3Lx + 2L^2), 0 \leq x \leq L$$
$$v_P(x) = \frac{-Fbx}{6EIL}(L^2 - b^2 - x^2), 0 \leq x \leq d$$
$$v_P(x) = \frac{-Fd(L-x)}{6EIL}[L^2 - d^2 - (L-x)^2], d \leq x \leq L$$
$$v_{total} = v_{M_D}(x) + v_P(x)$$

To Find F:

$$\begin{aligned} v_{M_0}(x = 2a) &= \frac{-M_0(2a)}{6EI(3a)}\left((3a)^2 - 3(3a)(2a) + 2(3a)^2\right) \\ &= \frac{-4}{9}\frac{M_0 a^2}{EI} \end{aligned}$$

$$\begin{aligned} v_F(x = 2a) &= \frac{-Fa(2a)}{6EI(3a)}\left((3a)^2 - a^2 - (2a)^2\right) \\ &= \frac{-4}{9}\frac{Fa^3}{EI} \end{aligned}$$

$$v_{total}(2a) = \frac{-4}{9}\frac{a^2}{EI}\left(Fa + M_0\right)$$

Recall:

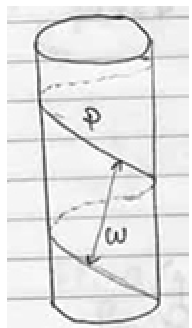
$$v_{total}(2a) = \delta$$

$$\frac{F}{k} = \frac{-4a^2}{9EI}\left(Fa + M_0\right)$$

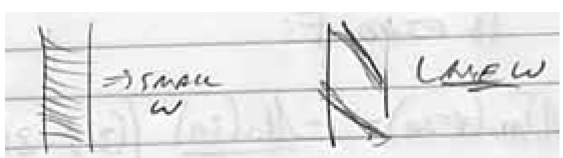
$$F\left(\frac{1}{k} + \frac{4}{9}\frac{a^3}{EI}\right) = \frac{-4a^2 M_0}{9EI}$$

$$F = \frac{\frac{-4a^2 M_0}{9EI}}{\left(\frac{1}{k} + \frac{4a^3}{9EI}\right)}$$

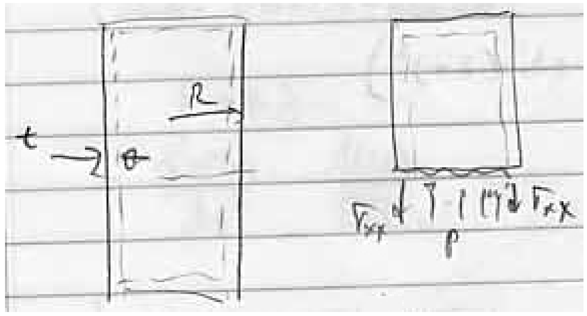
Example: Seamed Pressure Vessel



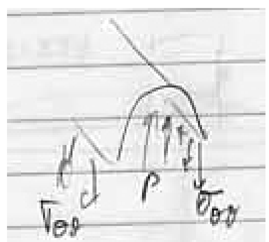
If max tensile stress on a seam is 0.8 max of the rest of the structure, what is max W?
Note:



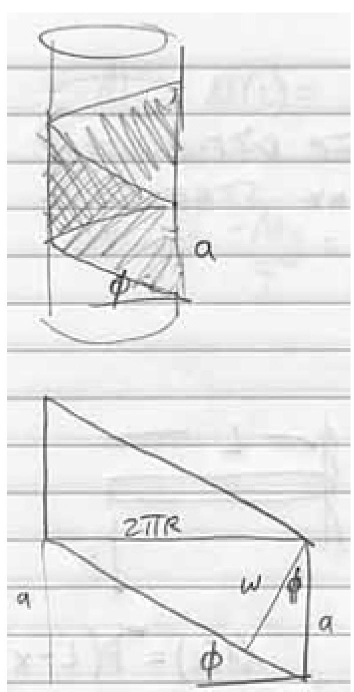
$$\sum F_x = 0$$
$$P\pi R^2 = 2\pi R t \sigma_{xx}$$
$$\sigma_{xx} = \frac{pR}{2t}$$



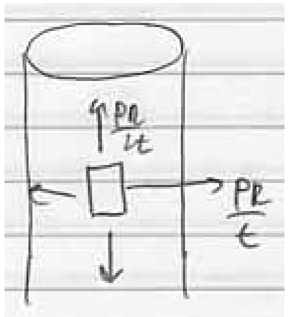
$$\sum F_\theta = 0$$
$$P(2R)\Delta x = \sigma_{\theta\theta}(2t\Delta x)$$
$$\sigma_{\theta\theta} = \frac{pR}{t}$$

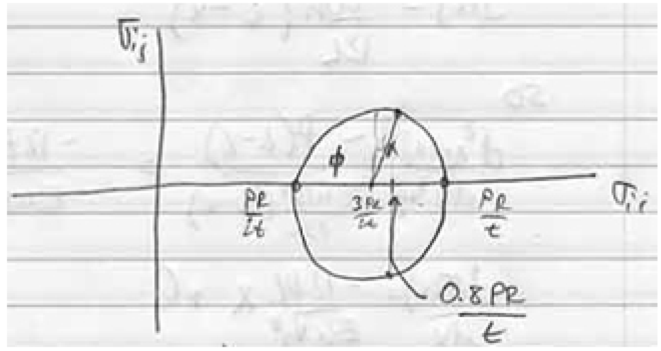


$$\tan\phi = \frac{a}{2\pi R}$$
$$\cos\phi = \frac{W}{a}$$
$$W = 2\pi R \sin\phi$$



Draw Mohr's Circle





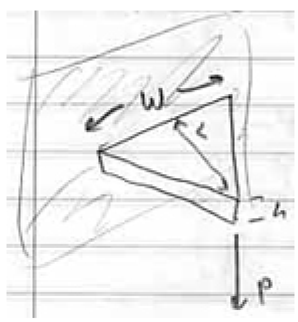
$$(2\phi) = 90 + \alpha$$

$$\sin\alpha = \frac{0.05\frac{pR}{t}}{.25\frac{pR}{t}} \Rightarrow \alpha = \sin^{-1}\left(\frac{1}{5}\right)$$

$$\phi = 45^\circ + \frac{\sin^{-1}\left(\frac{1}{5}\right)}{2}$$

$$W = 2\pi R \sin\left(45 + \frac{\sin^{-1}\left(\frac{1}{5}\right)}{2}\right)$$

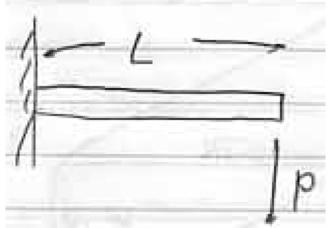
Example:



What is tip deflection?
What is max stress?

$$\frac{d^2v(x)}{dx^2} = \frac{M(x)}{EI(x)}$$

$$I = \frac{1}{12}b(x)h^3$$



$$M(x) = -P(L-x)$$

$$b(x) = \frac{W}{L}(L-x)$$

$$I(x) = \frac{wh^3}{12L}(L-x)$$

So:

$$\frac{d^2v(x)}{dx^2} = \frac{-P(L-x)}{\frac{EWh^3}{12L}(L-x)} = \frac{-12PL}{Ewh^3}$$

$$\frac{dv(x)}{dx} = \frac{-12PL}{EWh^3}x + c_1$$

$$v(x) = \frac{-6PL}{Ewh^3}x^2 + c_1x + c_2$$

At $x = 0$, $v = 0 \Rightarrow c_2 = 0$
At $x = 0$, $\frac{dv}{dx} = 0 \Rightarrow c_1 = 0$

So:

$$v(x) = \frac{-6PLx^2}{Ewh^3}$$

$$v_{tip} = v(L) = \frac{-6PL^3}{Ewh^3}$$

$$\sigma_{xx_{max}} = \frac{-My}{I} = \frac{P(L-x)\frac{h}{2}}{\frac{wh^3}{12L}(L-x)} = \frac{6PL}{wh^2}$$

Note: Stress along top is not a function of x.