2.001 - MECHANICS AND MATERIALS I
Lecture #18
11/13/2006
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Failure of Materials

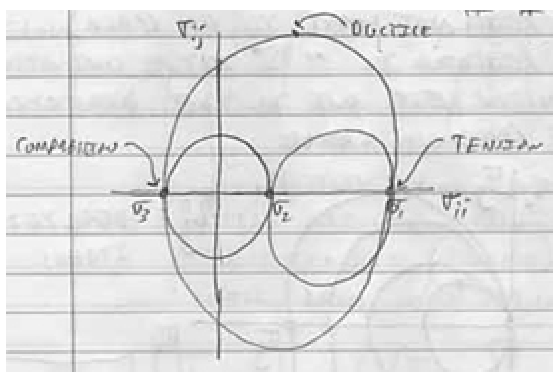
3-D:
x-y-z frame:

$$[\sigma\ ] = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix}.$$

Express in principal frame x'-y'-z'

$$[\sigma\ ] = \begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}.$$

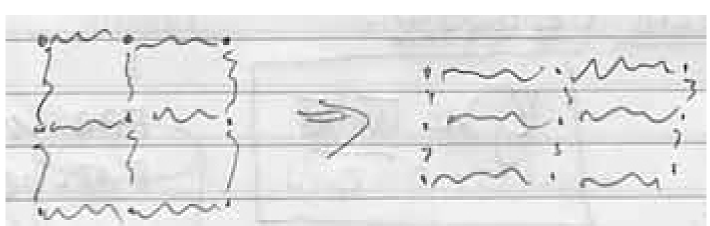
x'-y'-z': Principal Directions
$\sigma_1 > \sigma_2 > \sigma_3$: Principal Stresses



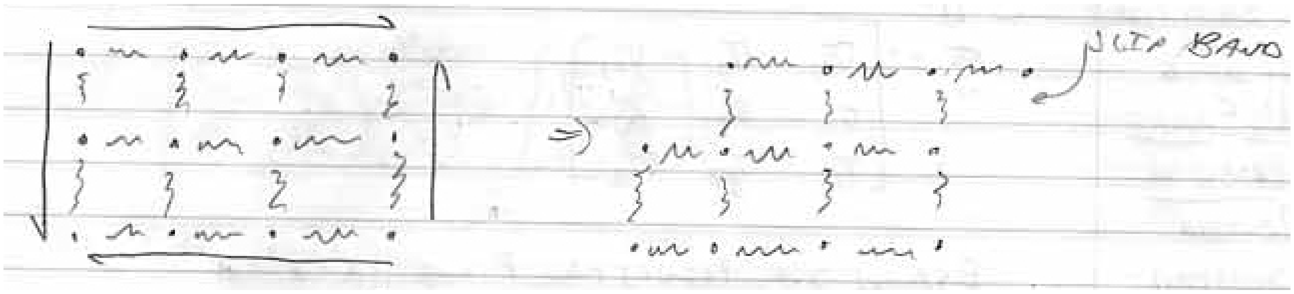
Ductile
Can be drawn out to a wire (Al, Cu)
Fail in shear $\Rightarrow$ yielding

Brittle
Will fracture
Fail in tension/compression

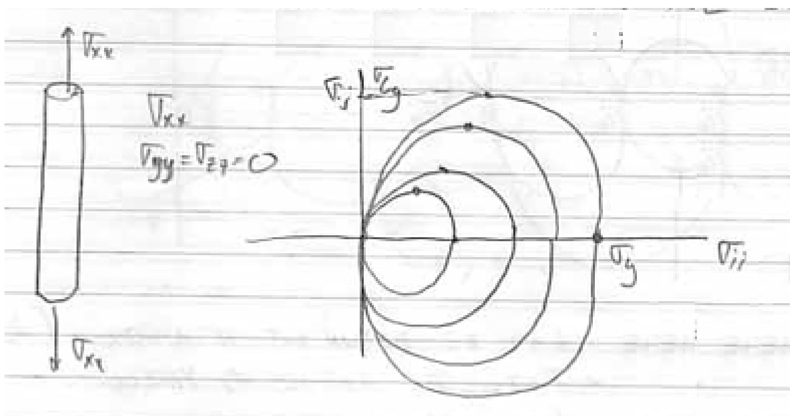
Recall description of interatomic bonds



What about loading in shear? (For ductile materials)



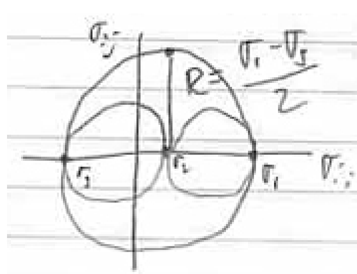
Bonds broke and were re-formed
This is called a "slip-and", it leads to yielding (bonds are reformed and do not want to go back). This yielding is 45° to the uniaxial tension test due to the direction of maximum shear.



$\tau_y =$ Shear yield stress

Note: $\tau_y = \frac{\sigma_y}{2}$.
How do you get a measure of yield under more arbitrary loading?

Tresca Yield Criterion



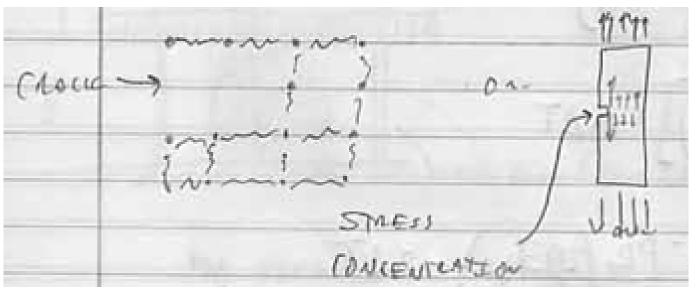
$$\tau_y > \frac{\sigma_1 - \sigma_3}{2} \text{ for no failure}$$

von Mises Yield Criterion: uses a measure of strain energy

$$\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2]} < \sigma_y \text{ for no failure}$$

Note: For uniaxial stress, $\sigma_{um} = \sigma_1$ (Same as Tresca)
For "pure shear" $\sigma_{um} \approx \sigma_{tresca}$ within about 15%

What about failure due to normal stress? (Brittle material ex. Chalk)



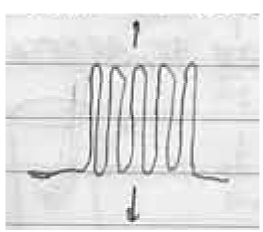
Brittle failure will occur at a stress concentration (defect). This stress concentration will decay within a couples times the size of the defect.

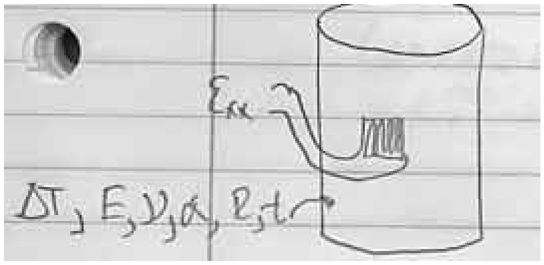
For no fracture:

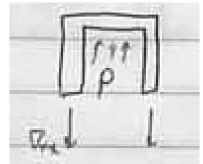
$$(\sigma_{normal})_m ax < (\sigma_{normal})_{maxallowed}$$

EXAMPLE: Measure pressure in a soda can

Strain gages: used to measure strain in a material, made of Si, Ge (Piezoelectric materials)

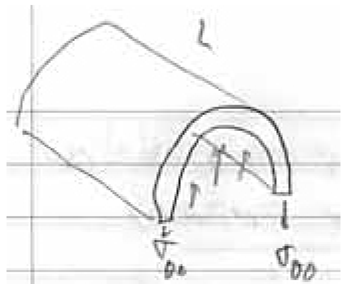






What is P?
    Use equilibrium.

$$p(\pi R^2) - 2\pi R t \sigma_{xx} = 0 \Rightarrow \sigma_{xx} = \frac{pR}{2t}$$



$$2pRL - \sigma_{\theta\theta}(2tL) = 0$$
$$\sigma_{\theta\theta} = \frac{pR}{t}$$
$$\sigma_{rr} \approx 0$$

Constitutive Relationship

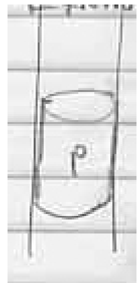
$$\epsilon_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{\theta\theta} + \sigma_{rr})] + \alpha\Delta T$$
$$\epsilon_{\theta\theta} = \frac{1}{E}[\sigma_{\theta\theta} - \nu(\sigma_{xx} + \sigma_{rr})] + \alpha\Delta T$$
$$\epsilon_{xx}^{full} = \frac{1}{E}\left[\frac{pR}{2t} - \nu\left(\frac{pR}{t}\right)\right] + \alpha\Delta T$$
$$\epsilon_{xx}^{measured} = -\epsilon_{xx}^{full} = -\frac{pR}{Et}\left(\frac{1}{2} - \nu\right) + \alpha\Delta T$$
$$p = \frac{-(\epsilon_x^{meas}x + \alpha\Delta T)}{(\frac{1}{2} - \nu)}\frac{Et}{pR}$$

What if you put the strain gage 90°?

$$\epsilon_{\theta\theta} = \frac{\Delta R}{R} \Rightarrow \text{ Similar}$$

What if you put the strain gage on at arbitrary θ?
    ⇒ Use strain transformation

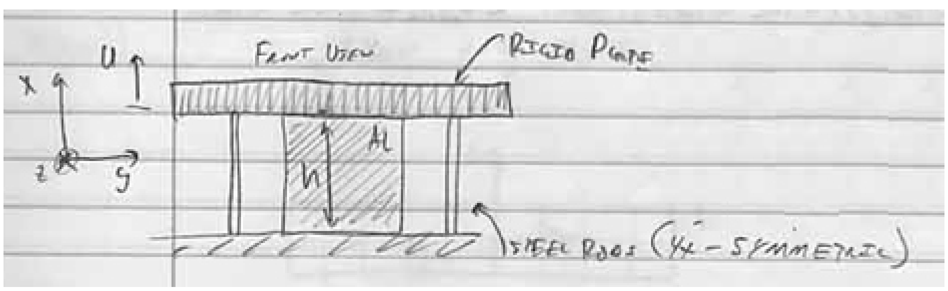
EXAMPLE:

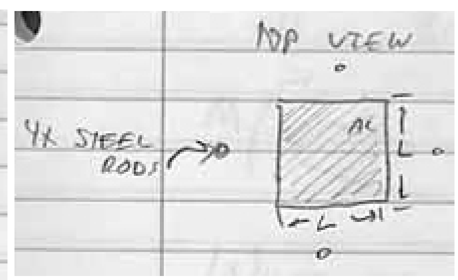


ΔT, p, $\sigma_{xx} = \frac{pR}{2t}$, $\epsilon_{xx} = ?$, $\sigma_{\theta\theta} = ?$, $\epsilon_{\theta\theta} = 0$

$$\epsilon_{\theta\theta} = 0 = \frac{1}{E}[\sigma_{\theta\theta} - \nu(\sigma_{xx} + \sigma_{rr})] + \alpha\Delta T$$

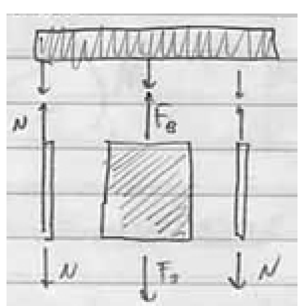
EXAMPLE:





$\alpha_{Al} \approx 24 \times 10^{-6} \frac{1}{^\circ C}$, $\alpha_{Steel} \approx 12 \times 10^{-6} \frac{1}{^\circ C}$, $\sigma_{y,steel} = 250$ MPa

FBD



$$\sum F_x = 0$$
$$-F - 4N = 0$$
$$F = -4N$$

Constitutive Relationships

$$\epsilon_{xx}^{steel} = \frac{1}{E_s}[\sigma_x^s x - \nu_s(\sigma_{yy}^s + \sigma_{zz}^s)] + \alpha_s \Delta T$$
$$\epsilon_{xx}^{Al} = \frac{1}{E_A}[\sigma_x^A x - \nu_A(\sigma_{yy}^a + \sigma_{zz}^a)] + \alpha_a \Delta T$$

Compatibility:

$$\epsilon_{xx}^{steel} = \epsilon_{xx}^{Al} = \frac{u_x}{h}$$
$$\delta^S = \delta^A = u_x$$

Result:

$$\sigma_{xx} = \frac{(\alpha_A - \alpha_S)\Delta T}{\left[\frac{1}{E_S} + \frac{4A_{bar}}{L^2 E_A}\right]}$$

For typical #'s $\Rightarrow \Delta T_{yield} \approx 100^\circ$