8.4   Brownian Motion

Warning: This project can become very large.

Compute and plot the paths of a set of many random "walkers"   that are confined by a pair of barriers at $+B$ and B (we assume that they all start at $x = 0$). A single random walk is computed by repeatedly performing the calculation

$$x_{j+1} = x_j + s \tag{17}$$

where s is randomly selected from $\pm s$ (for a small $s > 0$). For example, one such walker can be defined by ;

```
x(1) = 0;
for j = 1:N
x(j+1) = x(j) + s*(2*rand(1,1)-1);
end
```

This works for one walker, but expand this to allow for many walkers that will "walk" in parallel. Use vectorial calculations, not loops.

Consider what happens when a walker reachers the "boundary" at $\pm B$. There are several ways in which the barriers can "act":

a. Reflect: In this case, when the new position of a waker is outside the barriers, the walker is "bounced" back by the amount that it exceeded the barrier. If you plot the paths, you should not see any positions that are outside the barriers.

b. Absorb: In this case, if a walker hits or exceeds the wall positions, it is absorbed and its walk ends. For this case, it can be of interest to determine the mean lifetime of a walker (*i.e.,* the mean and distribution of the number of steps the "average" walker will take before being absorbed).

c. Partially absorb: This case is a combination of the previous two cases. When a walker encounters a wall, "a coin is flipped" to see if the walker is reflected or is absorbed.

d. Maybe you can think of some other interesting boundary condition? Perhaps something non-physical, like a teleportation of particles from the boundary to the interior? Or a periodic boundary condition, whereby the particle gets moved by $\pm 2B$ so that it is inside the domain again?

You will need logical indexing to deal with the boundary conditions in an efficient way.

What can one do with all the walks that have been generated? Compute statistics, of course, to answer questions like:

- What is the average position of the walkers as a function of time?

- What is the standard deviation of the position of the walkers as a function of time?

- Does the absorbing or reflecting boundary conditions influence these summaries?

- For the absorbing/partial-reflection case, a plot of the number of surviving walkers as a function of step number/time is a very interesting thing, particularly if graphically displayed. What about the "density" of walkers? Can you plot it?

- You can approximate the density of walkers as a function of time and space. This is a nice smooth function which can be expressed analytically. Can you show that the density you get is an approximation to that function?

- What happens if you start with a distribution of walkers, not all at $x = 0$? what happens to the distribution of walkers with time?

- What if your walk is biased so that the probability for left is not equal that for right? What are the answers to the questions above?

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