PROFESSOR: Our last class Yi is running from his home in New Jersey due to snow. So he couldn't fly in. But actually, now I'm learning a lot. It's a good way to run the classes going forward. I think. We may employ it next year. So Yi will present CV modeling for about an hour. And then Jake, Peter and myself, we will do concluding remarks. We will be happy to answer any questions on the projects or any questions whatsoever. All Right? So Yi, please. Thank you.

YI TANG: OK. I'm here. Hi everyone. Sorry I couldn't make it in person because of the snow. And I'm happy to have this opportunity to discuss with you guys counterparty credit risks as a part of our enterprise-level derivatives modeling. I run a Cross Asset Modeling Group at Morgan Stanley. And hopefully you will see why it's called Cross Asset Modeling.

OK, counterparty credit risk exists mainly in OTC derivatives. We have an OTC derivative trade. Sometimes you owe your counterparty money. Sometimes your counterparty owes you money. If your counterparty owes you money, on the payment date, your counterparty may actually default, and therefore, either will not pay you the full amount it owes you. The default event includes bankruptcy, failure to pay and a few other events. So obviously, we have a default risk. If our counterparty defaults, we would lose part of our receivable.

However, the question is before the counterparty defaults, do have any other risks? Imagine you have a case where your counterparty will pay you in 10 years. So he doesn't need to pay you anything. Then the question is are you concerned about counterparty risks or not? Well, the question is yes, as many of you probably know, it's the mark-to-market risk due to the likelihood of a counterparty future default. It is like the counterparty spike widens, even though you not need a payment from you counterparty.

If you were to sell, the usual trades to someone and someone may actually worry about that. So therefore the mark-to-market will become lower if the counterparty is spread wider. This is similar to a corporate bond in terms of economics. You owe a bond on the coupon payments date of the principal date counterparty can default. Of course, they can default in between also. But in terms of terminology, this is non-called counterparty risk. This is called issue risk.

So here comes the important concept credit valuation adjustment. As we know the counterparty is a risk. Whenever there's a risk, we could put a price on that risk. Credit valuation adjustment, CVA, essentially is the price of a counterparty credit risk. Mainly mark-to-market risks, of course, include default risk too. It is an adjustment to the price of mark-to-market from a counterparty default [? frig ?] model, the broker quote. So people know, there's a broker quote. The broker doesn't know the counterparty risk. A lot of our trade models do not know the counter party risk either, mainly because of we're holding it back, which I will talk about in a minute.

Therefore, there is a need to actually have a separate price of CVA to be added to the price for mark-to-market from counterparty default free model to get a true economic price. In contrast, in terms of a bond, typically there's no need for CVA because it is priced in the market already. And CVA not only has important mark-to-market implications, it is also a part of our follow three cap hold Not only change your valuation, but could impact your return on capital. Because of a CVA risk, the capital requirements typically is higher. So you may have a bigger denominator in this return RE, return on capital or return on equity.

CVA risks as you may know, has been a very important risk, especially since the crisis in 2008. During the crisis, a significant financial loss actually is coming from CVA loss, meaning mark-to-market loss due to counterparties' future default. And this loss turned out to be actually higher than the actual default loss than the actual counterparty default.

Again, coming back to our question, how do we think in terms of pricing a derivatives and the price, the CVA together with the derivatives. First of all, it adds some portfolio effect by the commodity and trade module effect. And the default loss or default risk can be different depending on the portfolio. And when people use a trade-level derivatives model, which is the [INAUDIBLE] what people would call a derivatives model, typically you price each trade, price one trade at time. And then you activate the mark to market together to get a portfolio valuation. So when you price one trade, you do not need to know there may be other trade [? hinging ?] on the same counterparty.

But for CVA or counterparty risk, this is not true. We'll go over some examples soon. This is the one application of what I call enterprise-level derivatives, essentially focusing on modeling the non-linear effects, non-linear risks in a derivatives portfolio.

Here's a couple of examples. Hopefully, it will help you guys to gain some intuition on the counterparty risks and CVA. Suppose you have an OTC derivatives trade, for instance like an IR swap. It could be a portfolio of trades. Let's make it simple. Let's assume the trade PV was 0 on day one. Of course, we assume we don't know anything about the counterparty credit risk. We don't know anything about CVA. This is just to show how CVA is recognized by people. So to start with again, the trade PV was 0 on day one, which is true for a lot of co-op trades. And then the trade PV became $100 million dollars later on.

And then your counterparty defaults with 50% recovery. And you'll get paid $50 million of cash. OK, so $100 million times 50% recovery. If the counterparty doesn't default, you eventually would get $100 million. Now he defaults, you get half of it, $50 million.

The question is have you made $50 million dollars or have you lost $50 million over the life of the trade. Anyone have any ideas? Can people raise your hand if you think you have made $50 million? Can I see the people in the class? I couldn't see anyone.

PROFESSOR: How do I raise this?

YI TANG: OK, no one thinks you made the $50 million. So I guess then, did you all think you have lost $50 million? Can people raise their hand if you think you have lost $50 million? OK, I see people. Some people did not raise your hand. That means you are thinking you are flat? Or maybe you want to save your opinion later? OK, so this is a common question I normally ask in my presentation. And I typically get two answers. Some people think they've made $50 million. Some people think they've lost $50 million. And there was one case, so I said OK, you know they're flat.

Now, this would look like a new interesting situation where no one thinks you made $50 million. I mean, come on, you have $50 million of cash in the door. And they don't think you have made $50 million. You have a $0 from day one. Now, you have $50 million. OK?

All right, anyway so for those of you who think you have lost money-- I don't know if it's a good idea [? Ronny-- ?] can someone tell us why do you think you lost $50 million? You went from 0 to positive $50 million. Why do you think you lost $50 million? Are we equipped to allow people to answer questions?

PROFESSOR: Yeah, I think if someone presses a button in front of them.

YI TANG: OK, so people chose not to voice your opinion?

AUDIENCE: It is because you have to pay to swap and you have to pay $100 million to someone on the other side of trade?

YI TANG: OK, very good. So essentially, you are saying hedging. That was what you are trying to get to? So you have a swap as 0 and you have an offsetting swap as a hedge. Is that what you are trying to say?

AUDIENCE: No. I'm saying that if you're the intermediary for a swap, then you have to pay $100 million on the other end. So if you're receiving 50 and paying 100, you have loss.

YI TANG: That's good. Right, so intermediary is right. And that's similar to a hedge situation also. So that's correct. That's the basically the reason for a dealer. Essentially, we are required to hedge. We're very tight on the limit. We actually would lose $50 million maybe on the hedge fund. When our trade went from 0 to a positive $100 million, our hedge would have gone from to 0 to negative $100 million. In fact, we receive only half of what we need to receive. And yet, we have to pay the full amount that we need to pay on the hedge side. Essentially, we lost $50 million.

But that's where the CVA and CV trading, CV risk management would come in. Again, CVA is the price of a counterparty credit risk. And now, if you hedge, right the underlying trade or whatever trade swap you if you hedge, you see the risk. Theoretically, you will be made whole if a counterparty default. So you would receive $50 million from counterparty, and theoretically you receive $50 million from the CV's desk if you hedge with CV desk.

Now, the second part is how do we quantify CVA. How much is the CVA? CV on the receivable, which we typically charge to the counterparty, essentially is given by this formula. MPE means mean positive exposure, meaning only our receivable sides when the counterparty owes us money, and time the counterparty ES [? call ?] spread, times duration. The wider the spread the more likely the counterparty will default, the more we need to charge on the CVA.

And the same thing is true for the duration. The longer the duration of trade is, there's more time for the counterparty to default so we charge more. Very importantly, there's a negative sign. Because CVA on the receivable side, is our liability. It's what we charge our counterparty. And there are some theoretical [? articles ?] that don't who define, that's OK for theoretical purposes. But practically, if you miss the sign things will get very confusing.

Now, here is more accurate formula for CVA. You know how the MPE side, we have SSI. So we can see to start with, there's an indicator function where this capital T is the final maturity of the trade or counterparty portfolio. This [INAUDIBLE] is the counterparty's default time, first default time. And if the [? pal ?] is greater than this capital T, essentially that means a default happens after the counterparty portfolio accrues. And therefore, we don't have counterparty risk. So that's what this indicator is about.

If the counterparty defaults before the maturity, that's when they will have counterparty credit risk. And there's a future evaluation of the counterparty portfolio right before one of our defaults. And this is how much collateral we hold against this portfolio. So the net receivable, the net amount, where the future value is greater than the collateral, is our sort of exposure, how much the counterparty would owe us.

And this is 1 minus R essentially is the default risk. So 1 minus r times the exposure essentially is the future loss given default. And beta essentially is a normal mock money market account for defaulting, and this is the expectation and the risk [? that you ?] measure. It looks simple. But if you get to the details, it's actually very complex maybe because the portfolio effected this option like payoff. If you recognize this positive sign here, essentially you recognize this is like optimal.

And so again, here is about some details of non-linear portfolio affects. First of all, we talk about often trades. In the previous example, you have one trade and went from 0 to $100 million. Counterparty defaults, you get paid $50 million, essentially, you lost $50 million. But what if you have another trade facing the same counterparty? Well, that's offsetting. When the first rate went from 0 to $100 million, the offsetting trade can goal from 0 to negative $100 million. And therefore if the counterparty were to default, you're going to have a 0 default loss. That's just one example of portfolio type because I'm offsetting trades. So therefore, in order to quote CVA, you've got to know all the trades you have facing the same counterparty. This is very different from a trade-level model where you only need to know one trade at a time.

There's also asymmetry of handling of the receivable, meaning assets versus the payable, meaning liabilities. And that's where the option might pay off, it comes about. Typically, roughly speaking, if we have a receivable from out counterparty, if the counterparty were to default we're going to receive a fraction of it. So we would incur default loss. However, if we have a payable to our counterparty, if the counterparty were to default, we more or less need to pay the full amount. We don't have a default game, per say. So this asymmetry is the reason for this option-like payoff we just saw previously.

And as you know, a counterparty can trade many derivative instruments across many assets, such as the [? engineer ?] effect, [? query ?] equities, a lot of [? types ?] also commodities and summaries of a mortgage. And then my group is responsible for the not only of the underlying exposure for CVA for capital as well as for liquidity, because multiple assets are involved and we need to model cross assets. So therefore, we named our group Cross Asset Modeling.

Furthermore, it is not only we have option-like payoff, which is [? modeling ?] here, we have an option essentially on a basket of a cross asset derivative trades. And the modeling becomes even more difficult. So that's when the enterprise-level will come in. And the enterprise-level model, which will come around even more later on, will need to leverage trade-level derivative models, and therefore, will need to do a lot of martingale really itself, martingale testing, resampling, interpolation.

So here's a little bit more information on the CVA. We have talked about assets or MPE CVA, essentially four our assets or receivable. In this formula, we have our default already in the first one. There is also, theoretically, a liability CVA. Essentially, it is the CV on the payable side, when the bank or when us having a likelihood of default. And this is a benefit for us, all right. So the formula is fairly symmetric, as you can recognize, except the default time or default event is not for the counterparty but for us. OK? And then the positive sign here became negative sign, essential to indicate this is a payable negative liability to us.

This is an interesting discussion first to default. We talked about how if the counterparty were to default, we more or less pay the counterparty full amount. So this is argumentatively used on the receivable side. So if we have a receivable, and if we were to default first, roughly speaking the counterparty would pay us close to the full amount. And there, some people start to think about OK, when we price CVA, we've got to know amount, counterparty in and of itself, which one is first to default. But my argument is that we do not need to consider that. And I have some reference for you guys to take a look if you are interested in this topic, but I'm not going to spend much time because we have lots to go over.

Now, here's another example. You have a trade, same as the previous trade. The trade PV was 0 on day one, and the trade PV becomes $100 million later on. This time of course the counterparty risk are properly hatched. Then the question is do you have any other risks. Does anyone want to try to tell us do you see any other risks? There are actually no several categories of risk we will have. I wonder if anyone would like to try to share with us your opinion. Sorry, I couldn't hear you. Yes?

AUDIENCE: Some form of interest rate risk.

YI TANG: Interest rate risk, ok, fine. OK fine, this is a market risk. Yes, you're right there is interest rate risk, but I did mention here that the market risk are properly hatched. So that means this interest rate risk of the trade will be handled by the hedge. What other risks?

AUDIENCE: Is there a key man risks? So if the trader that made the trade leaves and doesn't know about the--

YI TANG: Ah, OK

AUDIENCE: --portfolio?

YI TANG: That's a good point. Yeah, there is a risk like that. Yeah. Any other risks? OK. Let's go over this then.

I claim there is a cash flow liquidity funding risk. OK? Our trade is not collateralized. And then I claim we need funding for uncollateralized derivative receivables, meaning we are about to received $100 million in the future. We don't have it now. And I claim we actually come up with cash for it in many cases, in most cases, not every trade. Anyone have any idea of why when you are about to receive money, you actually need to come up with money? This comes back to the hedge argument similar to CVA.

Essentially, if you were to hedge your trade with futures or with another dealer which are typically collateralized. That means when you are about to receive $100 million, essentially you are about to pay $30 million on your hedge. In fact, you had to be futures that maybe mark to market, that means you need to actually really come up with $100 million cash. The same is true for collateralized trades. And there that's where the risk is. Because when you need to come up with this money and you don't have it, what are you going to do? You may end up like Lehman.

And there's also a contingent on the liquidity risk, meaning how much liquidity risk is dependent on the market conditions, how much interest rates changed, how much other market risk factor changes like that. And depending on the market condition, the liquidity may be quite different and you may not know beforehand. So that's the another challenge. [INAUDIBLE] If you turn the argument around, applying the argument to the payable and if you have uncollateralized payable, essentially you would have a funding or liquidity benefit. So one interesting thing to manage this liquidity risk essentially is to use and collateralized payable funding benefits to partially hedge the funding risk and collateralized derivatives receivables. There are a lot of other risks, for instance, [? payer ?] risk and equity capital risks.

Now here is one more example I'd like to go over with you guys on the application of CVA. This is about studying post options or post spreads. If you trade stocks yourself, you may have thought about this problem. I mean, either you can buy the stock outright or you sell put possibly with a strike lower than the current price. With that, you more or less have a similar pay out. Some people may offer you OK, if you see put, if your stock comes down, you're going to lose money. But you're going to lose money if you were to hold the stock outright also.

One of these strategies that is the sell put, if the stock is not good to you, and you're not participating the up side when the stock price increases significantly then you are not going to capture that price. But of course, one thing people can do is [? you continue to ?] sell put so they become like an income trade. So it's an interesting strategy. Some people say that selling put is like name your own price and get paid for trying it. And that's why we have this payments trade and Warren Buffet, Berkshire, so the long dated put for leading stock industry. In US, you pay your venture tax collected about four billion premium without posting collateral. Without posting collateral, that was very important.

This is something I actually was very involved in one of my previous jobs. This happened about, I think, around 2005, 2006. It's one of the biggest trades . And I was told when I was involved with this, this was one of the biggest cash outflow in the derivatives trades at that time, because Warren Buffet collected the premium without posting collateral. If he had agreed to post collateral during the crisis of 2008, he's going to post many billion of dollars of collateral. And one reason he had more cash than other people was he's very careful [INAUDIBLE] and I think I put a reference if you are interested and then you can essentially see the [INAUDIBLE] link [INAUDIBLE].

And what's interesting is that, there were quite a few dealers who are interested in this trade, but they know the size. And in a long-dated equity, I'd say, is not easy to handle, but I think of a lot of people were able to handle. To me, some people were not able to trade or enter this trade, not because they could not handle the equity. It's they could not handle the CVA compounded. First of all, we know there's a CVA. Essentially, we bought this option from Warren Buffett. Eventually, you may need to pay and at that time, he may default. So that's a regular CVA risk.

But there's also a rolling risk, meaning a more severe risk. You can imagine when the market sells off, Warren Buffet would actually owe us more money. Do you think in that scenario he will be more likely to default or less likely to default? He'll be more likely to default. That's where the term rolling risk comes in. When your counterparty owes you more and more money, that's when he's more likely to default. And that's even harder to model. And there's a liquidity funding risk which can also be long way, because as a dealer you may need to come up with [? deal ?] in [? order ?] to cash to pay Berkshire. Where do you get the money from? Specifically, people need to issue a debt to fund in a [? sine ?] secure way and essentially, you'll pay for quite a spread on your debt. That is essentially the cost of your liquidity of funding.

So what we did was, essentially, we charged Warren Buffett CVA and Vomer CVA, charge of the funding costs, some runway funding costs. Another challenge, of course, is that some dealers, I suspect, they could've priced CVA, but they do not have a good CV trading desk risk management to deliver risk management of CVA and funding. Once you have this position at hand, you have counterparty risks. But how do you hedge it? You charge Warren Buffet x million dollars for the CVA. If you don't do anything, when their spread widens, you're going to have a lot more CVA loss. So you need to risk manage that. Of course, you can do that with any hedge. But at any hedge, if we drill down to details, you suffer a fair amount of gasp risk. It's a lot like a bond. If you own a bond, you can buy a CDS protection on the same bond. More or less, you are hedged for a while a steady way.

But for a CVA, it's not. The reason for that is the exposure can change over time. One thing we tried at a time, essentially we sort of structured the credit linked note type of a trade. Essentially, you go to people who own or would like to buy Berkshire's bond. Essentially, you should tell them OK, we have a credit asset similar to Bershire's bond. If you feel comfortable with owning Bershire's bond, you may consider our asset which pays more [? coupon. ?] And the reason we were able to pay more coupon is we were able to part and held Berkshire a lot of money. And there's also a trough, people borrow protection thing that's involved, but I'm going to get that for the sake of time.

So then the question is the we tried [? a lot of ?] the money from Berkshire. Why would he want to do this trade? What would they think? So here's my guess. As you know, they have an insurance business. Then they wanted to explore other ways to sell insurance. So selling puts essentially is spreading insurance on the equity market. They sold like 10, 15 year maturity puts at below their spots. So then people can think, OK, what's the likelihood of a stock price coming down to below the current stock in 10, 15 years. Well, it happens, but it's not very likely. And they do have a day one cash inflow.

So essentially, I think one way Berkshire was thinking is that they thought low funding costs. If you read Warren Buffet's paper, essentially he's saying it's like 1% interest rate on a 10 year cash, or something like that. And it's very important to manage your liquidity well. They do not have any cash flow until the trade matures. So that's how they avoided the cash flow drain during 2008, even though they did suffer unrealized mark to market loss.

But what's interesting is that it during 2008, 2009, Berkshire did explore the feasibility of posting collateral. This started with no collateral posting. Hell, they wanted to post collateral. They actually approached some of the dealers saying oh, I want to post some collateral. Why is that? There's no free lunch.

So what happened was they were smart not to post collateral, but during the crisis they were spread widens. Everyone's spread widens. So Berkshire's spread widened. Then Warren Buffet owed more money. So guess what? The CVA hedging would require the dealer to buy more and more protection on Berkshire. When you buy more on someone, that will actually drive that person's, that entity's, credit spread even wider. So Berkshire essentially saw their credit spread widening a lot more than they had hoped for, than they had anticipated. And later on, they found out it was due to CVA hedging, CVA risk management.

That actually affected their bond issuance. When you have a high credit spread from CDS market, essentially the cash market may actually question may actually follow. And whoever would like to buy Bershire's bond would think twice. OK, if I have to buy this bond, if I ever have to buy credit protection, it's going to cost me a lot more money because of the spread widening. So therefore, I'm going to demand higher coupon on Berkshire's bond. that drives to their funding cost high. So they explored in different to post model.

Another thing of course is a very interesting thing to ask. Berkshire thinks they're making money and the dealer thinks they're making, which is probably true. But then the question is, who is losing money or who will lose money. Anyone has any ideas? I think there's probably a lot answers to this. My view is that essentially whoever needs to hedge, whoever need to buy but. If the market doesn't decline as much as much as you hoped for, essentially you'll pay for put premium and do not get the benefits.

Here's an interesting CV conundrum. Now, hopefully by this time, you guys fully appreciate the CVA risks and the impact of CVA. In terms of risk itself, in terms of magnitude, as I mentioned earlier being the crisis, 2008 crisis, which [? killed ?] among easily billions of dollars loss for some of the firms due to CVA, and that's more than the actual default loss. Now given you know the CVA, so if you trade with counterparty A, naturally you'll say you want to think OK, I want buy protection to hedge my CVA risk, to buy credit protection on A, from counterparty B. If you trade with counterparty B, you would have CVA against counterparty B. You would have a credit risk against counterparty B.

So what are you going to do? If you just follow the simple thinking, essentially you may think oh OK, maybe I should buy credit protection on B from counterparty C. But if you were to do that, then you have to continue on that. It becomes an infinite series. Infinite series are OK I'll say theoretically, but in practice I feel it's going to be very challenging to handle. So what would be a simple strategy to actually terminate this infinite series quickly?

Yeah this also has theoretical implications for CVA pricing. Sometimes we say, OK, I'm just not pricing this repeat [? replication, ?] use hedging instruments. Now, you have to use an infinite number of hedging instruments. That's going to impact your [? replication ?] modeling. So the way we would do it practically is to buy credit protection on A from counterparty B fully collateral, typically from a dealer. So however much money you owe from company B right away, they're going to post model. In a way, it's more or less similar to a futures content setting. That minimized the counterparty risk [INAUDIBLE]. So you can cut off this infinite series easily.

Here, I'd like to touch upon what I call enterprise-level derivatives modeling. We mentioned trade-level derivatives models. That is essentially, is just a regular model. When people talk about derivatives model, usually people talk about trade-level models. Essentially, you model each trade independently. Your model is price, mark to market or it's Greeks sensitivity. Then when you have a portfolio of these trades, essentially you can just aggregate their PV, their Greeks, through linear aggregation. Then essentially you get the PV of the portfolio.

But as we have seen already, that doesn't capture the complete picture. There are additional risks that require further modeling. One is non-linear portfolio risks. So essentially, these risks cannot be like a linear aggregation of the risks of each of the component trades in the portfolio. The example we have gone through is CVA, funding is of similar nature, capital liquidity are also examples. The key to handle this situation is to be able to model all the trades in the market and the market risk factors of a portfolio consistently so that you can handle the offsetting pay properly. Of course, we need to leverage the trade-level model essentially to price each individual trade as of today as of a future date.

What's interesting is that there's also feedback to the trade-level models. For instance, when we price a [INAUDIBLE] levels model of a very public trade. Now this [INAUDIBLE] level model we trade with a counterparty, let's say assumed [INAUDIBLE] we trade with a counterparty that's close to default. You know the trade-level model doesn't know about it's counterparty, about default.

The trading level will give you independent, the exercise boundaries, when do you need to cancel this model independent of the counterparty credit quality. That invites a question, when the counterparty is close to default, even if your model says OK, you should not exercise based on the market conditions, but shouldn't we consider the counterparty condition, credit condition. If the counterparty were close to default, if you cancel this off sooner essentially you'll eliminate or reduce the counterparty risk. This is actually interesting application and feedback between a trade-level model and the enterprise-level models.

So what we did was, in some of my previous jobs, what we did was actually figure out the counterparty risk in these trades, the major trades. Then essentially, we just held the underlying trading. if you were to cancel this trade, we have a benefited because we're going to reduce the CVA or even zero the CVA. So the CV trader would be able to pay the underlying trade. So therefore, the underlying model actually can take as equals rather than as part of the exercise condition modeling knowing if you cancel earlier you potentially can get additional benefits. This model may eventually be able to tell you to handle the risks more properly, market risks together with counterparty risks. This is roughly the scope and the application of the enterprise-level model.

This is actually a fairly significant modelling effort as was a significant infrastructure data effort. Essentially, it requires a fair amount of martingale testing, martingale [? recovery, ?] martingale interpolation and the martingale modeling. The reason for that is you have a trade model, and your trade model can model a particular trade accurately, and there's certain market modeling simulations of the underlying market or great PV.

But when you put a portfolio of trades together, now the methodology you use for modeling one trade accurate may not necessarily be the methodology you need to model all the trades accurately. Some of these require PV and some require simulations, but you need to put them together. Typically, we use to simulation. And that introduced numerical inaccuracies.

And the martingale testing will tell us are we introducing a lot of errors, martingale will suddenly essentially allow us to correct the errors. As you know, the martingale is a foundation of the arbitrage pricing. Essentially, martingale is something that will actually be able to enforce the martingale conditions in the numerical procedure, not only theoretically. Martingale interpolation modeling are other important interesting aspects if we have time we can [INAUDIBLE] There are different approaches for how to do it in a systematic way and still remain additional ways.

I'd like to quickly go over some of the example of martingale and martingale measures. I may need to go through this quite quickly due to the time limitations. But hopefully, you guys have learned all these already. This will hopefully be more like a review for you guys. So essentially, we are talking about a few examples.

What's the martingale measure forward qualified, forward LIBOR, forward public, forward the defense trade, forward CDS coupon. I would hope you guys would know the first few already. The for CDS [? for ?] coupon in my view is actually fairly challenging. For simplicity, I'm not considering the collateral discounting explicitly. That adds additional challenges but still we can address that.

So under the risk neutral measure, essentially for this y of p being the price of a traded asset with no intermediate cash flow. Essentially, that is y T over beta t is a martingale. This is essentially the entire collateral discount martingale lognormal theorem. It says for two traded assets with no intermediate cash flows, satisfying technical conditions, the ratio is a martingale. There's a probability measure corresponding to the numerical axis.

Therefore, naturally we have this composite. The forward arbitrage free manual essentially corresponding to [? a number error ?] of 0 small amount. Naturally, we can find this while p and at the PV ratio is a martingale. Again, it's just a ratio of two traded assets with no intermediate cash flow. From the definition of the full price, essentially the full price is a martingale under the forward measured.

Forward LIBOR-- this is the forward LIBOR-- essentially, it's a ratio of two zero for one bond. So naturally, we know it's a martingale under of the numerical axis. So essentially it's a forward measure up to the payment of the forward LIBOR. So this is the martingale condition. similarly, we can do this argument of the forward swap rate. Essentially, a forward swap rate is we can start with it like that the annuity you numeraire. And this is the forward swap rate, you essentially know is the difference of two zeros from one bond divided by annuity. And therefore we can conclude based on Harrison-Pliska theorem the forward swap rate essentially is the martingale under the annuity measure, with this annuity as the numeraire.

The same argument goes for the forward FX effect. Mainly the idea is or the pattern you probably have seen is, for any quantity you see if you can two assets and then use these two asset ratio to represent this in a quantity. So the forward FX essentially it's a ratio like this. This is nothing more than the interest rate parity.

From the stock you grow [INAUDIBLE].

You start with stock, you grow the domestic currency and then you grow the foreign currency. You get FX forwards. And FX forward is starting a martingale measure under the domestic forward measure. This is a simple technique do the change all the probability measure. It's roughly high number change of various measure and [INAUDIBLE] derivatives. You essentially start with, again, martingale, assuming this is martingale and there is a particular measure corresponding to the numeraire asset. And then this quantity is also a martingale and a different measure corresponding to a different numeraire asset.

One key point is when you change probability measure essentially you change the numeraire corresponding to the probability measure. And therefore essentially the important thing is we know the PV or the mark to market, but a tribute security is measured independent. It matters what mathematics you use if the traded security is going to mask the market price.

And therefore, you can price this security and the one measure or one numeraire. And then you can price again with another measure, another numeraire. They've got to be the same. Then naturally, you see there's a simple equation at the starting point to do the change of measure. If you just assume the change of variables, they essentially you get your change of measure as well as well [INAUDIBLE] usually. And if you worked on the VGA model, you'll probably recognize this change of measure which is used for the VGA model and under the old measure.

Now here's the this other heap, credit derivatives. Naturally, people would think OK, since the forward swap rate is a martingale and an annuity model, naturally people would think OK, then forward CDS part rate, it's like a forward swap rate. It's got to be a martingale under the risky annuity measure. So that's quite intuitive except there's one problem If the reference credit entity has zero recovery upon default. Then, this risky annuity could have a 0. And now we're talking about we want to use something that could be 0 as our numeraire. How do we resolve the technical mathematical problem. So that actually very interesting.

Schunbucher was the first person who published a paper on this model. I was just trying to do some work myself when I was working on VGA. I thought oh, it would've been nice to expand the VGA model to the other credit users. But then immediately I stumbled with this difficulty where when the recovery is 0, you're going to have a 0 in your numeraire, in your risky annuity. So Schunbucher, essentially, his idea was let's focus on survival measures, meaning we have a difficulty if a default happens and the recovery is 0. Now his idea is let's forget about that state. Let's not worry about that.

One immediate question people will as, if that's the case, the probability measure physical probability measure or risk neutral probability measure, then this survival formula are not equivalent because the survival probability measure knows nothing about the default event. So that they are not equivalent that's essentially you actually transform one mathematical difficulty to another one. Luckily, the second one turns out to be actually easier to solve.

So the starting point is again using Harrison-Pliska theorem. Essentially, you just need to identify like a numeraire asset, and the denominator assets. You identify two assets. You make a ratio and then those are a martingale. So essentially this is forward operating and forward annuity. If we have this indicator of the default time of a [INAUDIBLE] credit name, which is as a T, essentially this is like the premium leg of CDS. That's a traded asset. So therefore, we have in a martingale we have this. The subtlety as you probably can envision is going to come in when we do the change of probability measures.

OK, so we have talked about how are we going to find the martingale measure of a CBS par coupon or forward CBS par rate. This is a starting point of martingale model. Essentially, for any quantity you want to model you try to find its martingale measure. Once you find this martingale measure, you can do a martingale representation. And then often times you need to a change of a probability measure. So that all the term structure functions, a consequence of a variables are modeled in a consistent probability measure. So finding the martingale measure is the first point.

Survival probability measure, essentially, he just defined this with. You can define this [? rate ?] [? including ?] with derivatives. I want to define that essentially-- if you remember the previous formula-- you will have a martingale condition like this. [INAUDIBLE] The probability measures are not equivalent anymore, but yet they can still do change of probability measure. You need to separately model what going to happen when the default happens if you want to use this model.

Now, I'd like to move onto the second part martingale, martingale testing and martingale [? recovery ?] and interpolation. Martingale testing essentially given the previously model formula's conditions. Those are, by the way, just examples. There are a lot more. Essentially, you know that's what it should be theoretically you just test in your numerical implementation and see if those conditions are satisfied. That's the martingale test. Martingale resampling is we know most likely if you were to test, we're going to fail. This is not necessary for enterprise-level models but even for trade-level derivatives models. A lot of times, I think the martingale conditions are not exactly satisfied.

So one way to do that, is to correct that, correct this error. The rationale is essentially because of a numerical approximations. Whatever quantity we model essentially is not a true quantity. The true quantity we model essentially is some function of what we have an amount of. So therefore, you're selecting certain samples. Sometimes you can have a linear, log-linear function. This x 0 is what we have in our model, and then x is what we need to satisfy the martingale conditions. Essentially, in this Purdue case is very simple. You first of all, use the mean and then you would adjust the deviation. So therefore, given any quantity x 0, you can have hack it. You can force it to be any given mean. This mean, in our case, will be determined by the martingale condition.

The next interesting thing is martingale interpolation. Oh, I have a typo here. Sometimes you have elected enterprise-level model, for instance, you're more liable. You liable you have a different [? tenors. ?] When you have as a curve you know at any given time, there's a term structure. In the model, a lot of times we can model a few selected points.

But what if your model requires a term structure, a term that not in your model. So what people normally do is you do martingale, you do interpolation. So you have a one year liable and you have a 5 year liable. And then you need a three years. What do you do? You interpolate, for instance. But interpolation doesn't automatically guarantee martingale relationships. The martingale interpolation has a goal of automatically satisfying the martingale relationships, so we're particular with our interpolating. Actually, it turns out to be a [INAUDIBLE] The starting point is the martingale condition that I wrote out on the slide.

Essentially, this s and t are the calendar time. And this capital T is really like a term structure. You have a one year rate, two-year rate, five year interest rate, those term structures. How do we interpolate such that after interpolation the corresponding martingale conditions are satisfied. So here's what we do. We start with, let's say, capital T1. Capital T1, that's a point we model. We assume that one is probably martingale resampled and satisfies the martingale condition. This is a martingale for T capital 2. That also satisfies the corresponding martingale conditions. Our goal is to figure out T3. How do you do interpolation for the term T3 such that this T3 will satisfy it's own corresponding martingale condition.

If you do simple linear interpolation using T as independent variable, essentially, you are not going to achieve that. So the key is we need the chose the proper independent variable for the interpolation. Essentially, it's the previous time or times 0 quantity. So time s is before time T. Imagine time s will be zero So using the corresponding quantities at a time 0 as the independent variable, essentially, you can achieve that. It's still linear interpolation, it's just to use a different independent variable. Essentially, you can show that very easily. This is just simple algebra.

If you take the expectation, this one being martingale, this little t will become s. Then if you do expecting hear, the little t will become little s. And therefore if you combine these two, a lot of terms will actually cancel. Essentially, you will be left with this martingale at time s and the T3, meaning this is the martingale target of this particular term . And that turns out to be the expectation of this quantity. So it's a very simple linear simple algebra. You guys can figure it out if you want to. So this one essentially guarantees the interpolated quantity for the values that satisfy all the conditions of the martingale target. Of course, you need to know the martingale part.

If you don't know, that's a different story. Then you need to do something else. Specifically, time 0, for instance, is what the market tells us Often time we do a big time assumption. So whatever assumption on time 0 you make, in you dynamic model, you automatically satisfy the needed martingale condition.

This is just a brief introduction of how we do the martingale modeling. This LIBOR market model, as you guys probably have learned already, there's different forms of BGM as the initial form. And then I'll [? James ?] [? Shidian ?] came with another form. And in terms of a general martingale model, what we'll do typically is we start to find the martingale one. And we know a forward LIBOR is a martingale in its own format. Then we know we can use martingale representation. Under certain technical conditions rise, the diffusion process can be represented by Brownian motion. That would assume log-normal just for example. We don't have to, we can use CV, with [INAUDIBLE] all the activity. The starting point is martingale and dividing the martingale measure and then perform martingale representation. Essentially, you get this stockcastic, eventually, picture.

They need a change a measure or change numeric. Because this one essentially say for particular liable, you have a Brownian motion and a different measure. So that has a limited usage. A lot of the derivative trade, IR trade essentially, it's the same with entire U curve. So you need to make sure you model the entire U curve consistently. So therefore you have to change to the probability measure so that everything is specified in the same common measure. Of course, you can have a choice which one you want to use as common measure. Through a simple change of numeraire, essentially, we can get a stockcast equation like this. We have a Brownian motion. Right, we have Brownian motion with a correlation like this.

So this is essentially a market model in a general form, with full dimensionality meaning one Brownian motion per term of a libel. So that's the full dimensionality.

PROFESSOR: Yi?

YI TANG: And then you need to do-- Yeah, hi.

PROFESSOR: Can you wrap up because we need a bit of time for questions.

YI TANG: Oh you need me to end. It's all right I can actually wrap up now. If you want to.

PROFESSOR: Sure. OK, any conclusions?

YI TANG: Well, OK. The conclusion is the following thing. There is a need for enterprise-level models to handle non-linear portfolio effects and we need to leverage our trade-level models. By doing so we do employ martingale testing, martingale resampling, interpolation. And not only we need that CVA, but we also need that for funding liquidity capital risks which are very critical risks. And people have started paying more and more attention to these risks, especially since after the prices. Because of time limits, I'm not going to be able to finish another example. But if you like, you can take a look on page 22 off the slides. Hopefully, [? Basilly ?] can still get it to you guys. Thank you guys.

PROFESSOR: Thank you Yi. We will publish the slides probably later tonight so please take a look. So to wrap up, let's me see. I want to bring up,

PROFESSOR 2: That's OK. Probably if it's the course website, that's fine.

PROFESSOR: I did add a few topics which were used last year for final paper for interest in the document which is on the website. So take a look. Basically, the themes there where mostly Black-Scholes or more advanced models or manipulation of Black-Scholes equation. There was a very interesting work on statistical analysis of commodities commodit data. So if somebody's up for it, that would be very interesting. And there were a few numerical in Monte Carlo projects. So any questions?

PROFESSOR 2: Yeah, so actually we were planning to give you a bit more time to ask your questions. But since we have five minutes, I think maybe I'd like to ask you to just think about what we learned this term. So Peter can add on what we think in on the mathematics and also those applications, and in conjunction while you're doing the final paper, just focusing on the new things you think that you learned and what did you like to explore in the next stage of your research. So I think probably we don't have a lot of time for lots of questions. But if you have any questions, this will be a good opportunity to ask about the paper or the course. Peter you want to make some comments?

PETER: Sure. I'd just like say that I think this course was a very challenging course for most of you and that was, I guess, our intention. And I really respect all the hard work and effort everyone put into the class. And in terms of the final paper, we will be looking at your background and look for insights that demonstrate what you've learned in the course. And I've already reviewed several papers. I'm very pleased with the results. So I think everyone's done a great job. This course, I think, is intended to provide you with the foundations of the math for the financial applications as well as an excellent introduction and exposure to those applications. I think you'll find this course valuable over the course your careers, and look forward to contributing insights with questions you might have following the course. I'm sure the other faculty feel the same way. We want to be a good resource for you now and afterwards.

PROFESSOR: Very, very well put. So please feel free to contact us. And please stay in touch. All the contact details are on the website. We plan to have a repeat of this class next year. So please, tell your friend or stop by next year, which will be the next fall. It will not be exactly the same. We will try to make it slightly different, but it will be close.

PROFESSOR 2: If you have any suggested topics, you feel you haven't been exposed to and would like to know more, send us email if you can. I think one of the values of this class is we can bring in pretty much everyone from the frontier in this industry to give you some insights of what's going on.

PROFESSOR: Please take care of you on the website, this is important. And that's all.

PROFESSOR 2: OK. Thank you for your participation this semester.

[APPLAUSE]

PROFESSOR: And thank Yi for the pleasure.