

Lecture 7: Value At Risk (VAR) Models

Ken Abbott

Risk Management's Mission

1. To ensure that management is fully informed about the risk profile of the bank.
2. To protect the bank against unacceptably large losses resulting from concentration of risks
3. In other words:

NO SURPRISES

- Two analogies:
 - Spotlight
 - coloring book

Methodology

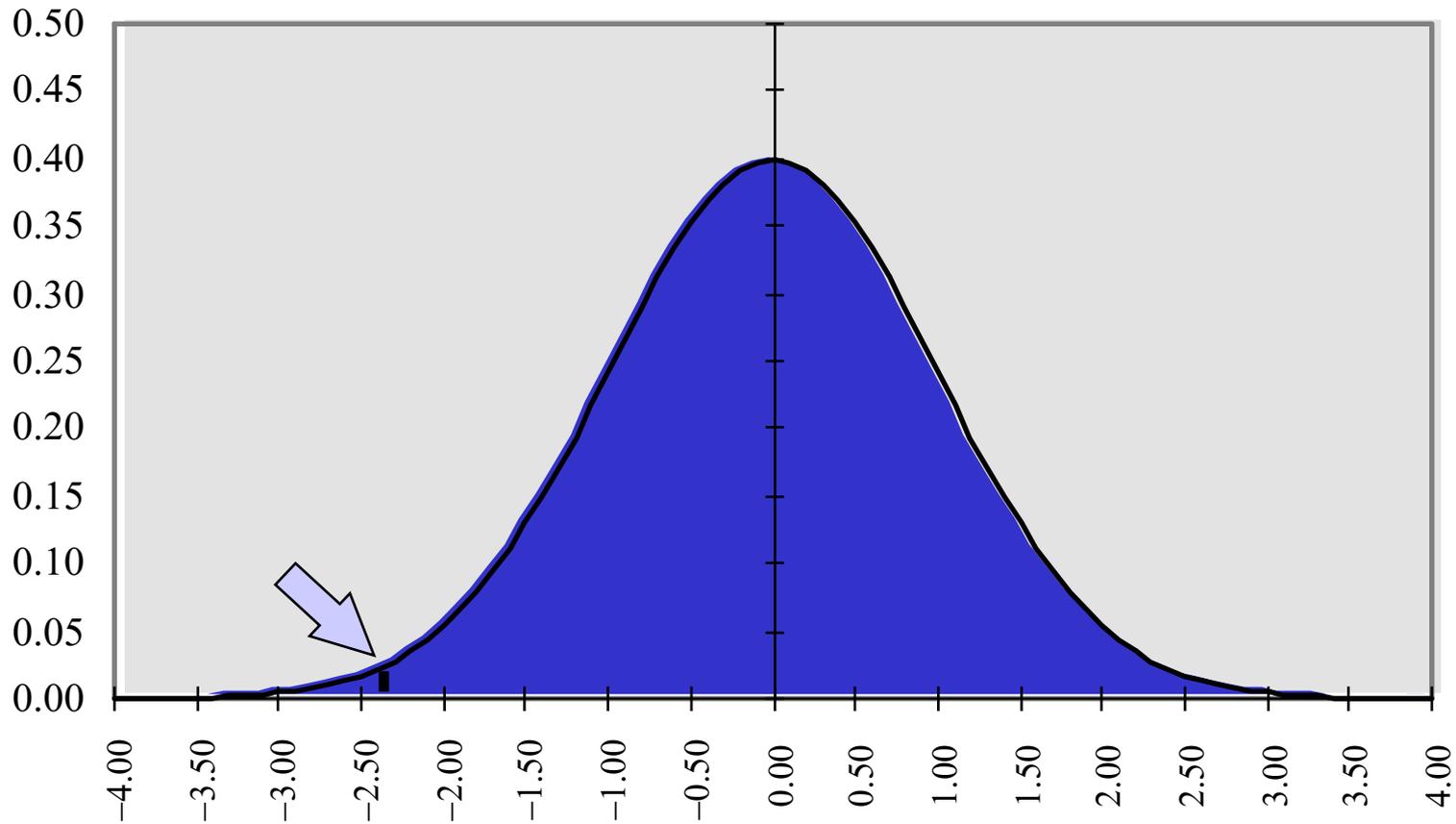
Different Methodologies

- Example of one-asset VaR
 - Price-based instruments
 - Yield-based instruments
- Variance/Covariance
- Monte Carlo Simulation
- Historical Simulation

Variables in the methods

1. Interest rate sensitivity – duration, PV01,
2. Equity exposure
3. Commodity exposure
4. Credit – spread duration
5. Distribution/Linearity of price behavior
6. Regularity of cash flow/prepayment
7. Correlation across sectors and classes

Methodology: What Are We Trying to Calculate?



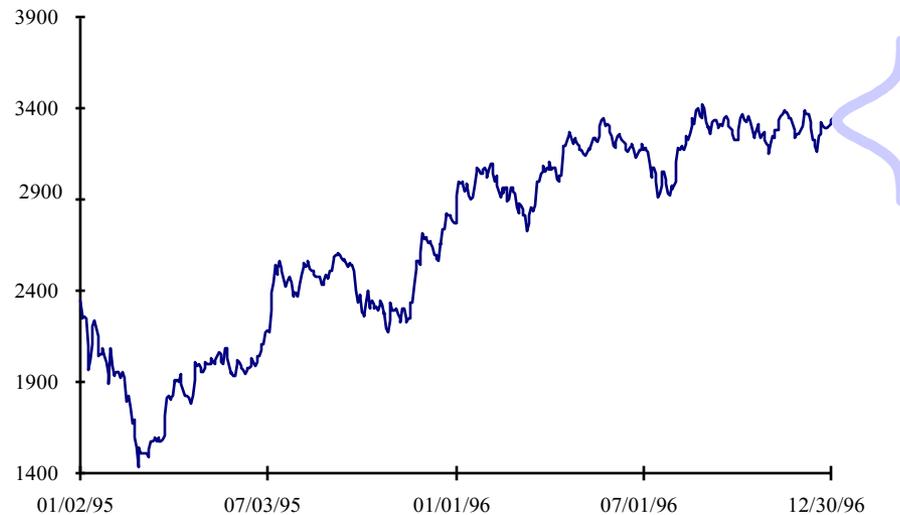
We want to estimate the worst 1% of the possible outcomes.

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No investment decisions should be made in reliance on this material.

Methodology: VaR Concepts

Date	IPC
01/02/95	2354.24
01/03/95	2278.47
01/04/95	2261.73
01/05/95	2273.1
*	*
*	*
*	*
12/25/96	3298.47
12/26/96	3310.89
12/27/96	3327.2
12/30/96	3346.95
12/31/96	3361.03

Consider one time series, the Mexican
IPC

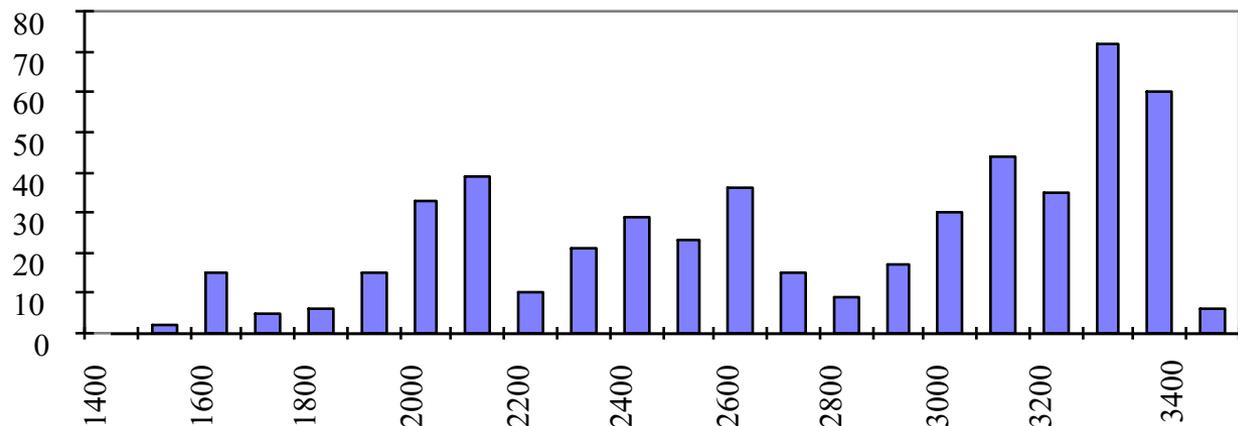


We want to know how much this market could possibly move against us, so we know how much capital we need to support the position .

Methodology: Why We Use Returns

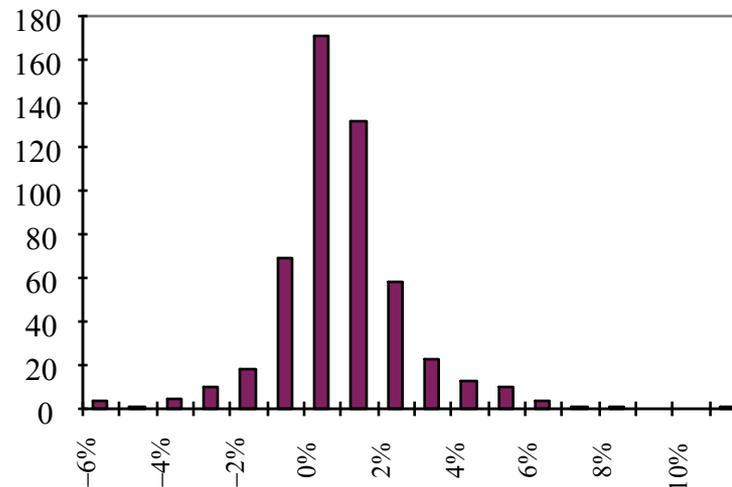
- **Most financial time series follow *random walks*, which means, among other things, that the best estimate of tomorrow's value is today's value.**
- **Since random walks are not bounded, predicting the future path is difficult if we focus only on the levels.**
- **A frequency distribution of IPC levels from 1995-1996 illustrates the difficulty:**

Frequency Distribution of IPC Levels: 1995-1996



Methodology: Why We Use Returns

- Consider the returns of the IPC over the same period, where returns are defined as the percentage change in the index:



- Here, the frequency distribution falls into a familiar pattern.

Methodology: Estimating Volatility

- Once we have a time series of returns, we can gauge their relative dispersion with a measure called *variance*.
- Variance is calculated by subtracting the average return from each individual return, squaring that figure, summing the squares across all observations, and dividing the sum by the number of observations.
- The square root of the variance, called the *standard deviation* or the *volatility*, can be used to estimate risk.
- In a normal distribution, 2.33 * the standard deviation represents the largest possible movement 99% of the time (1.64 * the standard deviation for 95%).

Methodology: Estimating Volatility

- Mathematically, variance is:

$$\sum_{i=1}^n \frac{(x_i - \bar{x})^2}{(n-1)}$$

- The standard deviation is the square root of this term.
- The Excel functions for these two are `var()` and `stdev()`

Methodology: Using Volatility to Estimate Value at Risk

- The variance of the daily IPC returns between 1/95 and 12/96 was 0.000324
- The standard deviation was 0.018012 or 1.8012%
- $2.33 * 1.8012\% = 0.041968$ or 4.1968%
- We can conclude that we could expect to lose no more than 4.1968% of the value of our position, 99% of the time.

Methodology: Using Volatility to Estimate Value at Risk

- This means that an investment in the IPC of MXP 100 investment would suffer daily losses over MXP 4.2 only about 1% of the time.
- In fact, the IPC lost more than 4.2% 8 times since 1/1/95, or about 1.5% of the time.
- While this figure is approximately accurate, it illustrates a problem VaR has in certain markets, that it occasionally underestimates the number of large market moves.
- This problem, while frequent at the security or desk level, usually disappears at the portfolio level.

Methodology: Review of One Asset VaR

1. Collect price data
2. Create return series
3. Estimate variance of return series
4. Take square root of variance to get volatility (standard deviation)
5. Multiply volatility by 2.33 by position size to get estimate of 99% worst case loss.

Methodology: Caveats

- Longs vs. Shorts: sign is important
 - simple for equities
 - requires thought for FX
- One-sided vs. two-sided confidence intervals
- Bad data
- Percentage changes vs. log changes

Methodology: Fixed Income

- Fixed income instruments require an adjustment to this method.
- This is because time series generally available for fixed income securities are *yield* series, while we are concerned with *price* behavior.
- The adjustment requires expressing the volatility in of basis points and the position in terms of sensitivity to a 1 basis point movement in yields.

Methodology: Fixed Income

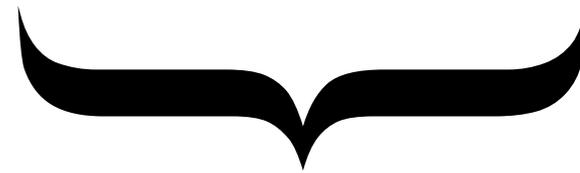
Equities, Foreign Exchange, Commodities:
position * σ_{price} * 2.33 (or 1.64 for 95%)

Fixed Income:

position * PV01 * close * σ_{yield} * 2.33 * 100



position sensitivity to a one
BP movement in yields



potential movement in
yields measured in BP

Methodology: Fixed Income

Example

- Consider a USD 100 position in 10 year UST:
 - Vol of the percentage changes in UST yields since 1/1/03 = 1.312%
 - DV01 = change in price at 12/31/96 yield if yields were to increase by 1 basis point
 - (Duration may also be used here)

Methodology: Fixed Income

Example

- Price at yield of 4.644% = 100
- Price at yield of 4.654% = 99.92077
- DV01 = 99.920765 - 100 = 0.07923 per \$100
- This sensitivity changes with the level of yields, but provides a good approximation

Methodology: Duration vs. DV01

- Duration measures the weighted average time to a security's cash flows, where the weighting is the cash flow.
- Duration also shows the percentage change in price per change in yield.
- DV01 provides a similar measure, but often per 1 million of face value.
- Bond traders think in DV01's; portfolio managers think in terms of duration.
- Either measure is effective but **BE CAREFUL OF THE UNITS**. This is one of the easiest errors to make!

Methodology: Duration & DV01

Examples

- UST Example:
- Use Excel PRICE() function
- Remember to divide coupon/yield by 100 (4.644% = .0644)
- Assume redemption at par
- Note similarity of PV01 to Duration

Basis	Day count basis
0 or omitted	US (NASD) 30/360
1	Actual/actual
2	Actual/360
3	Actual/365
4	European 30/360

	R	S	T	U				
1042	Date	12/27/2006		Price @ Yield = Coupon				
1043	Maturity	12/27/2016		100.00000				
1044	Coupon	0.04644		Price @ Yield = Coupon + .0001				
1045	Yield	0.04654		99.920765				
1046	Redemption	100		Implicit PV01 (100 - Price @ plus 1bp)				
1047	Frequency	2		0.0792349				
1048	Basis		1	Duration @ Yield = Coupon				
1049				7.9272577				
1050								
1051				=PRICE(S1042,S1043,S1044,S1045,S1046,S1047,S1048)				

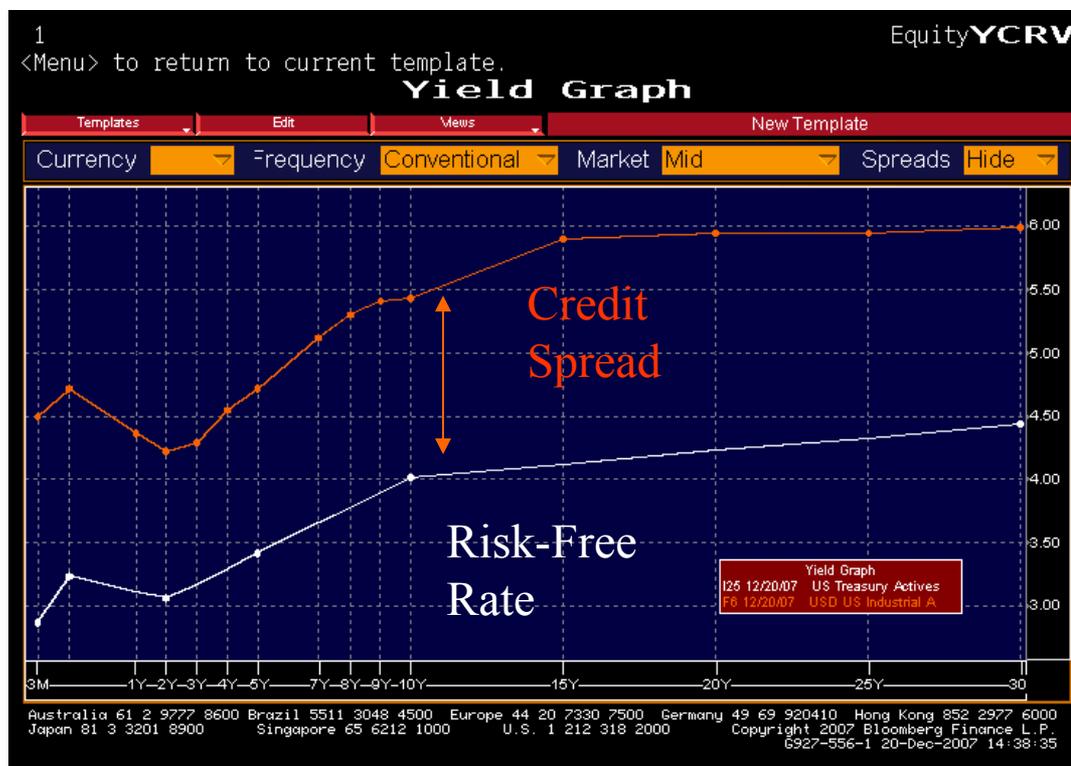
Methodology: Putting It All Together

- Position details:
 - Size: USD 100
 - Maturity: 10 Years
 - Vol: 0.01312
 - DV01: -.07923 (*per 100*)
 - 12/27/06 Close = 4.644%
- $$\text{VaR} = -.07923 * 0.04644 * .01312 * 2.33 * 100$$
$$= 1.12479$$

Spread PV01

- For credit-risky securities, we should distinguish between interest rate risk and credit risk
- The credit spread takes default (and recovery) into consideration
- We usually consider these separately
- Often, we assume $PV01 = CSPV01$
 - If recovery=0, then this is true
 - Otherwise, it is not
- There are different sources for spreads
 - Calculated
 - CDS
 - Asset swap spreads

Spread PV01



Methodology:

Adding Additional Assets

- The notion of covariance allows us to consider the way assets' prices behave with respect to each other.
- Technically:

$$\sum_{i=1}^n \frac{(x_i - \bar{x})(y_i - \bar{y})}{(n - 1)}$$

Methodology: Covariance

- What does this mean?
 - It gives an indication of how far one variable is from its mean when we observe another variable a certain distance from its mean.
 - In other words, it says how much (and in which direction) y moves when x moves.
 - It provides a measure for every variable with respect to every other variable.

Methodology: Portfolios

- Some Basic Statistical Principles:

variance(a+b)=variance(a)+variance(b)+2*covariance (a,b)

$$\sigma_{a+b}^2 = \sigma_a^2 + \sigma_b^2 + 2\sigma_{a b}$$

a	b	a+b	Calculations-----	
-0.22	-0.39	-0.61	Var(a)	0.333059
-0.53	-0.28	-0.81	Var(b)	0.469281
-0.03	0.64	0.61	Cov(ab)	0.349660
-1.05	-0.76	-1.81	Var(a)+Var(b)	
0.87	1.67	2.54	+2*cov(ab)	1.501660
-0.21	0.12	-0.09		
0.00	0.06	0.06		
-0.20	0.34	0.14	Var(a+b)	
-0.11	-0.32	-0.43	estimated directly	1.501660
0.00	0.12	0.12		
0.27	0.09	0.36		
-0.23	-0.18	-0.41		
1.26	1.34	2.60		

Methodology: Covariance

- Why is this important?
 - If we know the variances and covariances of all of the securities in a portfolio, we can assess the risk of the entire portfolio.
 - We can also assess the risk of any subportfolio.
 - This is the basis of much of modern portfolio theory.

Methodology: Correlation

- How is correlation different from covariance?
 - we can scale covariance to get correlation:

$$\frac{\text{Cov}(a,b)}$$

$$\sigma_a \sigma_b$$

- covariance is not unit free
- correlation is an index of linearity

Methodology: Correlation

- Does it matter which one we use?
 - As long as we adjust for units, no.
- Why use one or the other?
 - intuition is easier for correlation
 - calculations are easier with covariance
 - If we know covariances, we also know the correlations, but NOT vice-versa.

Methodology: More Facts

- So far, we have examined portfolios with only one “unit” of each asset.
- Most portfolios hold several shares, several bonds, or several contracts.
- variance of (xa) where x is units (shares, contracts, bonds) $= x^2 \text{ var}(a)$
- variance of $(xa+yb)$ $= x^2 \text{ var}(a) + y^2 \text{ var}(b) + 2xy\text{cov}(ab)$

More Facts (continued)

- This can be extended:
 - $\text{var}(a+b+c) = \text{var}(a) + \text{var}(b) + \text{var}(c) + 2\text{cov}(ab) + 2\text{cov}(ac) + 2\text{cov}(bc)$
 - $\text{var}(xa+yb+zc) = x^2\text{var}(a) + y^2\text{var}(b) + z^2\text{var}(c) + 2xy\text{cov}(ab) + 2xz\text{cov}(ac) + 2yz\text{cov}(bc)$
 - $\text{var}(a+b+c+d) = \text{var}(a) + \text{var}(b) + \text{var}(c) + \text{var}(d) + 2\text{cov}(ab) + 2\text{cov}(ac) + 2\text{cov}(ad) + 2\text{cov}(bc) + 2\text{cov}(bd) + 2\text{cov}(cd)$
 - d) $\text{var}(xa+yb+zc+wd) = x^2\text{var}(a) + y^2\text{var}(b) + z^2\text{var}(c) + w^2\text{var}(d) + 2xy\text{cov}(ab) + 2xz\text{cov}(ac) + 2xw\text{cov}(ad) + 2yz\text{cov}(bc) + 2yw\text{cov}(bd) + 2zw\text{cov}(cd)$

Simplifying the Arithmetic

- Obviously, this gets messy, very fast.
- If we are to extend this to portfolios containing many assets, we need to find a way to simplify the calculations.
- To do this we need two new concepts, one simple and one fairly complicated.
 - Covariance and Correlation matrices
 - Using linear (matrix) algebra

Covariance/Correlation Matrices

Correlation

	M0A0	GA10_	C0A0	SPX	IGOV	MXEF	C0A1	H0ND	H0HY	H0A0	VIX
M0A0	1.00	0.84	0.62	(0.17)	0.35	(0.10)	0.74	0.11	0.01	0.01	0.13
GA10_	0.84	1.00	0.56	(0.31)	0.20	(0.26)	0.84	(0.11)	(0.19)	(0.18)	0.24
C0A0	0.62	0.56	1.00	0.24	0.67	0.33	0.84	0.60	0.56	0.58	(0.30)
SPX	(0.17)	(0.31)	0.24	1.00	0.56	0.82	(0.01)	0.62	0.66	0.67	(0.69)
IGOV	0.35	0.20	0.67	0.56	1.00	0.64	0.45	0.73	0.69	0.71	(0.53)
MXEF	(0.10)	(0.26)	0.33	0.82	0.64	1.00	0.06	0.66	0.70	0.71	(0.63)
C0A1	0.74	0.84	0.84	(0.01)	0.45	0.06	1.00	0.20	0.14	0.17	(0.11)
H0ND	0.11	(0.11)	0.60	0.62	0.73	0.66	0.20	1.00	0.96	0.95	(0.53)
H0HY	0.01	(0.19)	0.56	0.66	0.69	0.70	0.14	0.96	1.00	0.99	(0.51)
H0A0	0.01	(0.18)	0.58	0.67	0.71	0.71	0.17	0.95	0.99	1.00	(0.52)
VIX	0.13	0.24	(0.30)	(0.69)	(0.53)	(0.63)	(0.11)	(0.53)	(0.51)	(0.52)	1.00

Covariance

	M0A0	GA10_	C0A0	SPX	IGOV	MXEF	C0A1	H0ND	H0HY	H0A0	VIX
M0A0	0.71	1.67	0.95	-0.66	0.86	-0.62	1.04	0.21	0.03	0.04	2.15
GA10_	1.67	5.52	2.39	-3.36	1.39	-4.33	3.31	-0.59	-1.47	-1.38	10.72
C0A0	0.95	2.39	3.30	2.00	3.56	4.28	2.55	2.55	3.30	3.48	-10.38
SPX	-0.66	-3.36	2.00	21.29	7.56	27.24	-0.09	6.76	9.87	10.25	-60.56
IGOV	0.86	1.39	3.56	7.56	8.62	13.58	2.20	5.03	6.55	6.90	-29.84
MXEF	-0.62	-4.33	4.28	27.24	13.58	51.57	0.69	11.12	16.25	16.75	-86.30
C0A1	1.04	3.31	2.55	-0.09	2.20	0.69	2.80	0.81	0.74	0.93	-3.42
H0ND	0.21	-0.59	2.55	6.76	5.03	11.12	0.81	5.58	7.29	7.39	-24.07
H0HY	0.03	-1.47	3.30	9.87	6.55	16.25	0.74	7.29	10.43	10.58	-31.44
H0A0	0.04	-1.38	3.48	10.25	6.90	16.75	0.93	7.39	10.58	10.95	-33.05
VIX	2.15	10.72	-10.38	-60.56	-29.84	-86.30	-3.42	-24.07	-31.44	-33.05	366.22

M0A0	Mortgage Master	IGOV	USD EM Sovereign Plus	H0HY	US Original Issue HY
GA10	UST Current 10yr	MXEF	MSCI EM Equity	H0A0	HY Master II
C0A0	US Corp Master	C0A1	US Corporates AAA	VIX	Vol
SPX	S&P 500	H0ND	US HY Non-Distressed		

Covariance Matrices

2 x 2	CAD	CHF
CAD	0.000037	-0.000018
CHF	-0.000018	0.000321

3 x 3	CAD	CHF	DEM
CAD	0.000037	-0.000018	-0.000017
CHF	-0.000018	0.000321	0.000257
DEM	-0.000017	0.000257	0.000227

4 x 4	CAD	CHF	DEM	ESP
CAD	0.000037	-0.000018	-0.000017	-0.000001
CHF	-0.000018	0.000321	0.000257	0.000159
DEM	-0.000017	0.000257	0.000227	0.000143
ESP	-0.000001	0.000159	0.000143	0.000142

5 x 5	CAD	CHF	DEM	ESP	FRF
CAD	0.000037	-0.000018	-0.000017	-0.000001	-0.000012
CHF	-0.000018	0.000321	0.000257	0.000159	0.000231
DEM	-0.000017	0.000257	0.000227	0.000143	0.000198
ESP	-0.000001	0.000159	0.000143	0.000142	0.000140
FRF	-0.000012	0.000231	0.000198	0.000140	0.000192

Notice that if we sum the items in the matrix we end up with the sum of the variances + 2 x the sum of the covariances.

We can use this to our advantage.

Covariance Matrices

- If a portfolio has one unit of each security whose prices are tracked in the covariance matrix, the portfolio variance is the sum of the items in the covariance matrix.
- This rarely happens in the real world.
- We have to find a way to deal with this.

Correlation, Covariance & Time

- The one-day time frame makes using correlation matrices less theoretically ambiguous
- Question of correlation stability over time
- Correlations tend to “swing” from neutral to directional when markets under stress
- Short time frames mean linear approximations less problematic

Using Matrix Algebra

- Matrix (linear) algebra is used to summarize arithmetic calculations involving rows and columns of figures.
- It can be thought of as arithmetic shorthand.
- It is easily performed in spreadsheets using the MMULT() and TRANSPOSE() functions.

Using Matrix Algebra

- A course in linear algebra is well beyond the scope of this training program,
- However, what you need to know to do variance/covariance analysis is relatively simple.
- It requires only the MMULT() and TRANSPOSE() functions.

Using Matrix Algebra

- Assume we have (\$100) in CAD/USD, a (\$50) in CHF/USD and (\$25) in DEM/USD.
- We arrange our spreadsheet like this:

	A	B	C	D	E	F	G	H	I	J
1										
2	-100	-50	-25		0.000037	-0.000018	-0.000017			-100
3					-1.8E-05	0.000321	0.000257			-50
4					-1.7E-05	0.000257	0.000227			-25
5										
6	=MMULT(TRANSPOSE(A2:C2),MMULT(E2:G4, I2:I5))					1.691875	Portfolio Variance			
7						1.300721	StDev (sqrt of variance)			
8						3.0306798	2.33* StDev			

- By doing this we simultaneously perform all of the arithmetic described earlier.

Using Matrix Algebra

	A	B	C	D	E	F	G	H	I	J	
1											
2	-100	-50	-25		0.000037	-0.000018	-0.000017			-100	
3					-1.8E-05	0.000321	0.000257			-50	
4					-1.7E-05	0.000257	0.000227			-25	
5											
6	=MMULT(TRANSPOSE(A2:C2),MMULT(E2:G4, I2:I5))					1.691875	Portfolio Variance				
7						1.300721	StDev (sqrt of variance)				
8						3.0306798	2.33* StDev				

- Several things to remember:
 - Must use Ctrl-Shift-Enter to enter matrix functions instead of just enter
 - number of positions must equal number of rows and columns in matrix

Interpreting the Results

	A	B	C	D	E	F	G	H	I	J
1										
2	-100	-50	-25		0.000037	-0.000018	-0.000017			-100
3					-1.8E-05	0.000321	0.000257			-50
4					-1.7E-05	0.000257	0.000227			-25
5										
6	=MMULT(TRANSPOSE(A2:C2),MMULT(E2:G4, I2:I5))					1.691875	Portfolio Variance			
7						1.300721	StDev (sqrt of variance)			
8						3.0306798	2.33* StDev			

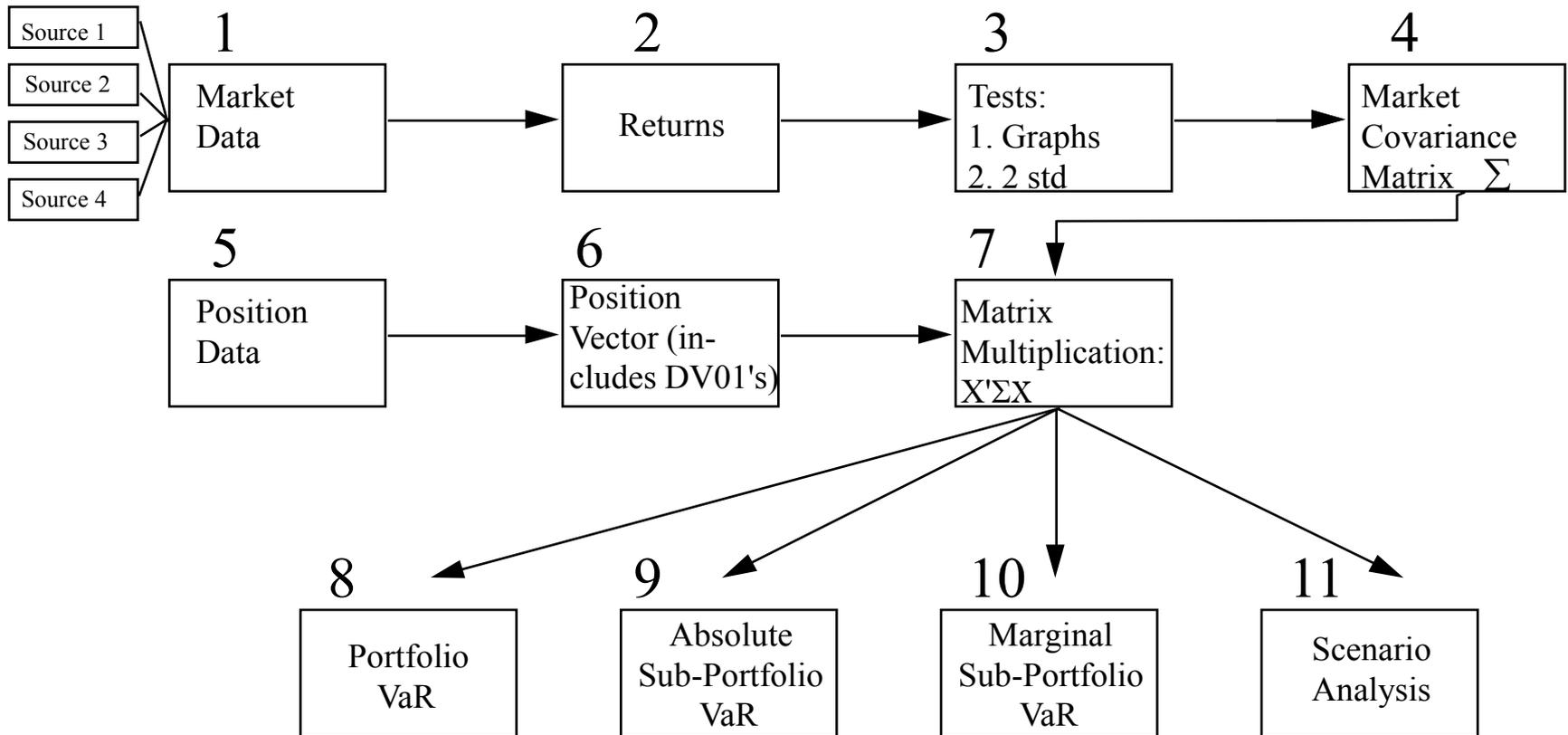
- The Excel function generates the *variance*. We must take its square root to get the standard deviation (volatility). Once we have the portfolio standard deviation, we multiply it by 2.33 to get the 99% value at risk.

The Big Picture

- Elements for Variance/Covariance calculations
 - data collection
 - the calculation of returns
 - data testing
 - matrix construction
 - positions and position vectors
 - matrix multiplication
 - capital calculation
 - interpretation

Flow Diagram

Variance/Covariance Analysis



Assumptions

- Getting Data
- Time period
- Weighting
- Bucketing
 - uncorrelated assets
 - related assets
- Gaps in Data
- Updating frequency
- Intervals for differences

Getting Data

- Price & security data sources
 - DRI (Fame)
 - Datastream
 - WWW
 - Bloomberg
 - Internal
- Covariance matrix sources
 - Riskmetrics
 - Bloomberg
- Credit info
 - Creditgrades/Riskmetrics
 - KMV
 - Ratings Agencies
 - Bloomberg

Time Period Coverage

- How far to go back
- Data availability
- Change in pricing regimes
 - 2008 Crisis
 - Brazil in 1995
 - Introduction of the Euro
- Existence of market anomalies
- Possible for traders to take advantage

Exponential Weighting

Different Weighting Schemes

- Unweighted
- Exponential

Exponential Weighting

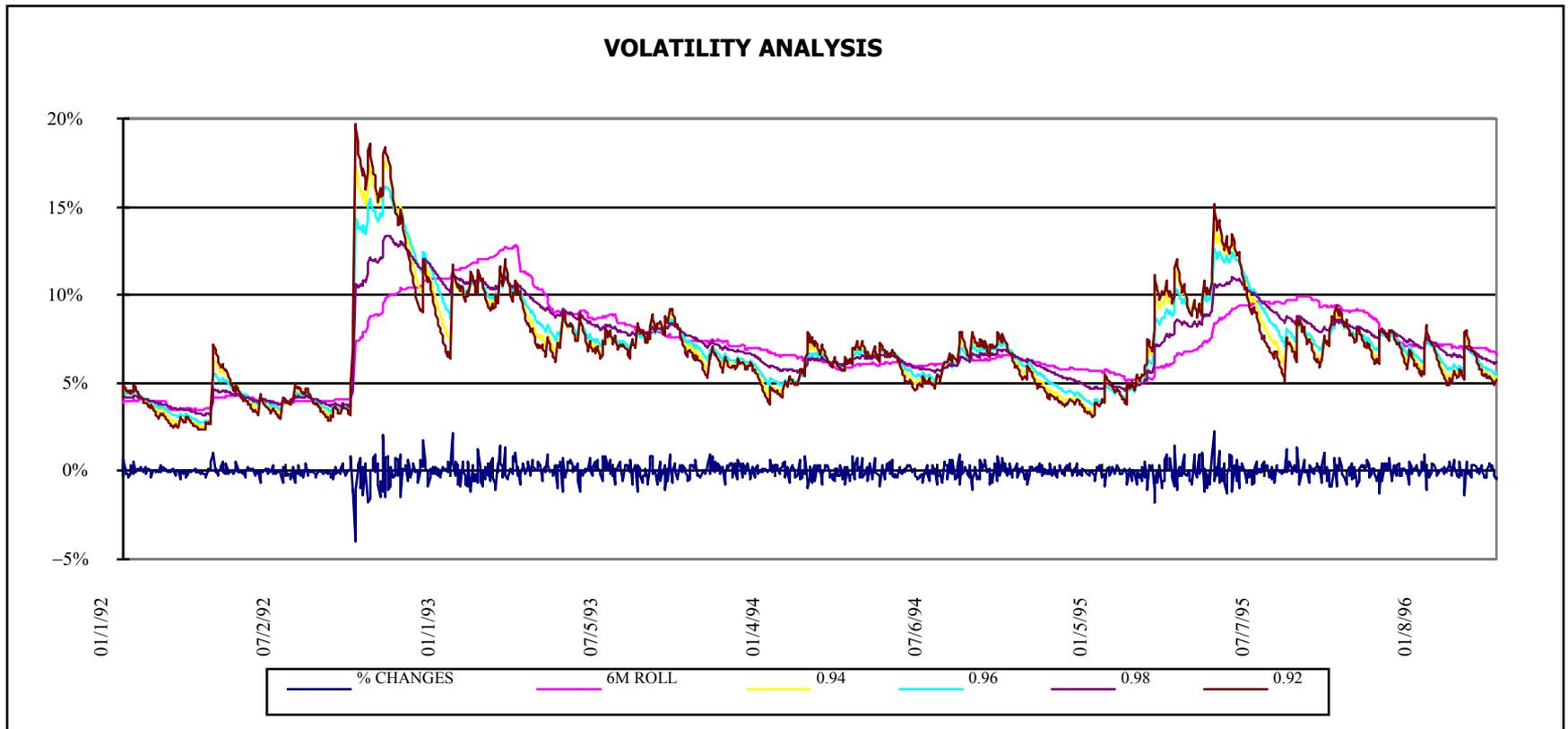
- The choice of a weighting coefficient has major implications
- can be more difficult to implement
- more difficult to check
- can have a major effect on variances and covariances
- in many ways, rather arbitrary
- However can smooth vol as positions age
- Acts in accordance with heuristic decision theory

Exponential Weighting

- weights more recent observations more heavily, declines exponentially
- need to estimate ω (weighting factor)
- equation for covariance usually of form:

$$\sum_{i=1}^n \frac{\omega^i (x_i - \bar{x})(y_i - \bar{y})}{\sum \omega^i}$$

Exponential Weighting



Bucketing

- We can't have a bucket for every position
- To accommodate this, we "bucket"
- By doing this, we reduce "granularity"
- If we do it right, it causes no decrease in accuracy
- Sometimes, we have to bucket the "uncorrelated assets" with a certain vol

Updating Frequency

- Daily vs. Weekly vs. Monthly
 - Really a question of your assumptions about the markets
 - How often do things move?
 - changes in vol common in s/t eurodeposits and Japanese rates
 - changes in correlation structure difficult to estimate

Percentage Changes vs. Log Changes

- The decision about which one to use is based primarily upon computational convenience.
- The only real problem with using percentage changes is that it theoretically allows rates and prices to become negative.

Differencing Interval

- Daily vs. Weekly vs. Monthly
 - What data is available?
 - Is the matrix used for trading or positioning?
- Can it be scaled up or down?
 - technically yes, but...
 - asynchronicity issues
 - serial correlation issues

Technical Issues

- Units
 - Are positions recorded in millions?
 - Are FX positions recorded consistently?
 - same base currency
 - correct sign (long or short base currency)
 - Fixed Income sign

Incorporating Fixed Income

- The previous example assumed that the positions were either FX, commodity or equity positions.
 - For these types of assets we measure *prices*.
 - The positions size x the largest price move expected (2.33 x standard deviation) directly tells us our expected worst loss.
- In fixed income markets, we measure *yields* .

Incorporating Fixed Income

- To include fixed income positions, we must multiply the position by something to reflect their *price* sensitivity to *yield* changes.
- This is done through the use of *DV01* (value of a basis point) or *duration* in the manner described earlier.
- The adjustment should be made in the position vector.

Absolute VaR

- What is the absolute VaR of each bucket?
- Two ways to calculate
 - take individual position sensitivity and multiply by position **vol** (not variance)
 - use position vector, but successively zero out all positions but the one of interest

Absolute VaR

Essentially, you are considering portfolios with only one asset:

Position Sensitivity Vector - Portfolio		Position Sensitivity Vectors: Bucket 1	Bucket 2	Bucket 3	Bucket 4	Bucket 5	Bucket 6	Bucket 7
10		10	0	0	0	0	0	0
20		0	20	0	0	0	0	0
35		0	0	35	0	0	0	0
-23		0	0	0	-23	0	0	0
40		0	0	0	0	40	0	0
100		0	0	0	0	0	100	0
-67		0	0	0	0	0	0	-67

One uses these vectors successively to calculate VaR.

Marginal VaR

- Why?
 - Primarily to see how an individual position contributes to portfolio VaR
 - Illuminates natural hedges
 - Shows efficacy of hedge portfolios

Marginal VaR

Position Sensitivity Vector - Portfolio	Marginal VaR Vectors: Bucket 1	Bucket 2	Bucket 3	Bucket 4	Bucket 5	Bucket 6	Bucket 7
10	0	10	10	10	10	10	10
20	20	0	20	20	20	20	20
35	35	35	0	35	35	35	35
-23	-23	-23	-23	0	-23	-23	-23
40	40	40	40	40	0	40	40
100	100	100	100	100	100	0	100
-67	-67	-67	-67	-67	-67	-67	0

Here, one compares each VaR with the original VaR to see the marginal risk contribution of the position.

Historical Simulation

- Historical simulation simply marks the portfolio to market using the rates and prices observed over a certain historical period; or changes in terms of standard deviations
- These P&L figures are then ranked and the order statistic derived by taking the n^{th} worst outcome
- n is a function of significance (usually 1%) and how much data is available

Historical Simulation

- Four Major Strengths:
 - Simple
 - Intuitively appealing
 - Nonparametric
 - Aggregation easy

However, Some Weaknesses

- Weaknesses
 - Need lots of data for good sample
 - May need historical data for things for which there is little history
 - How do you assume new products would behave?
 - Assumes stationarity

Performing HS

- What are we really doing?
 - Can be thought of as limited Monte Carlo
 - same general idea & process architecture
 - nonparametric
 - Back testing current portfolio

Steps

1. Collect position data
2. Bucket to risk indices
3. Calculate sensitivities (and deltas, gammas)
4. Collect historical data
5. Calculate returns
6. "Shock" portfolio with returns data
7. Calculate P&L figures
8. Rank P&L figures and get order statistic

Collect Position Data

- 90% of risk management is knowing what you own and where you own it
- Decisions must be made about granularity
- Synchronous data is important
- Derivatives present certain problems
 - greeks
 - format of position data (notional/MTM value)

Bucket to Risk Indices

- Again, level of granularity is important
- Several choices of bucketing methods
 - equal VaR
 - duration weighting
 - hedge positions
- Units are important
 - sizes of positions
 - potential confusion with fixed income

Calculate Sensitivities

- For equities, FX, commodities, very easy
- For fixed income: $\text{pos} * \text{DV01} * \text{close} * 100$
- Convexity not a big problem
- For options, delta or delta & gamma
 - for daily VaR, delta is probably OK
 - need gamma for long holding period
 - may need for volatile markets

Collect Historical Data & Calculate Returns

- Collect data
 - Sources
 - Problems
 - Missing data
 - Choice of time period
 - Frequency
- Calculate Returns
 - % change or log change

Shock Portfolio & Calculate P&L

- Now that we have sensitivities and shocks, we can generate the daily P&L numbers
- These numbers have some noise built in due to convexity and gamma
- These figures represent the hypothetical P&L from holding this portfolio over the last n days

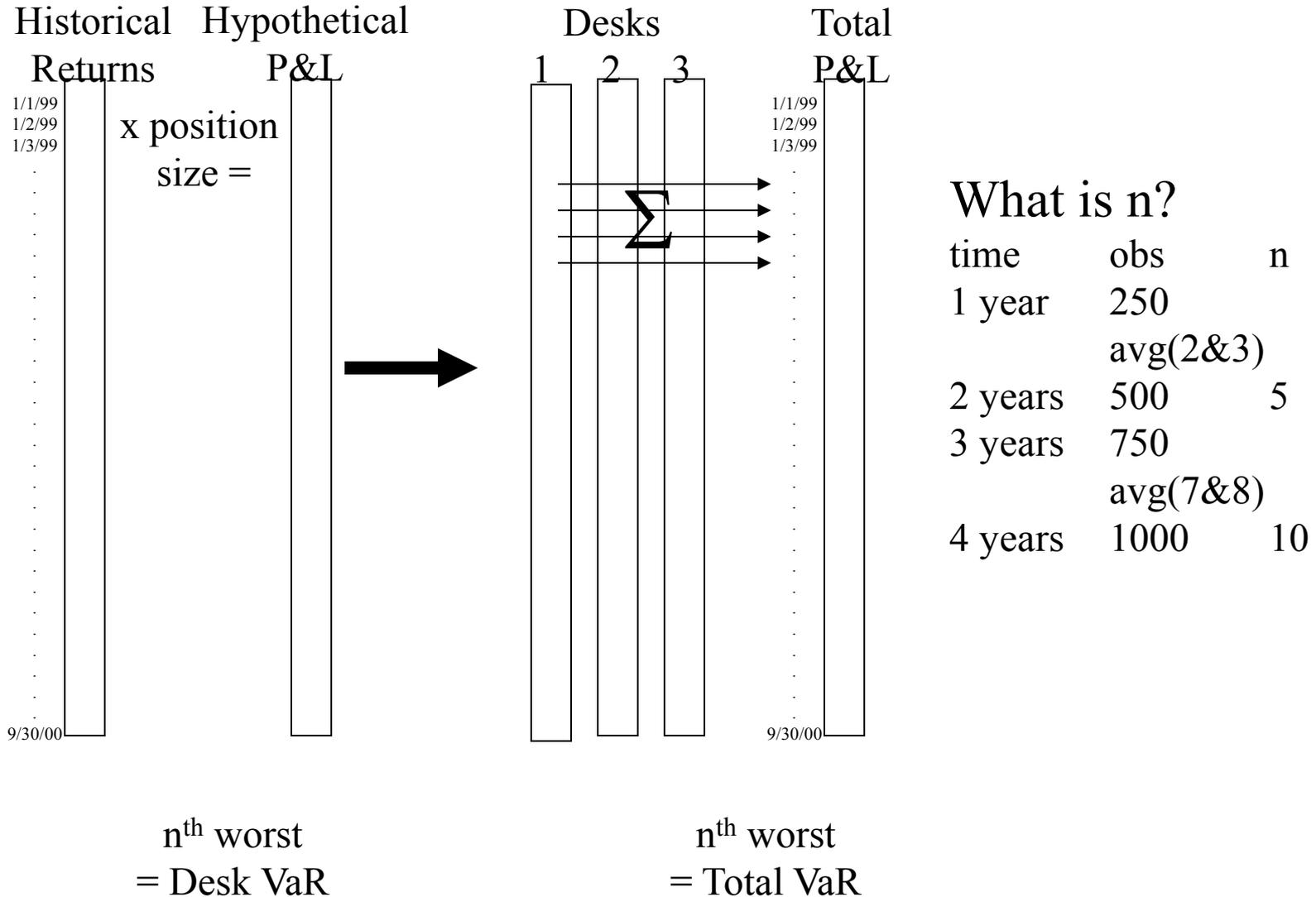
Rank P&L

- Simply sort the P&L figures
- Take n th worst for order statistic
 - Probably makes sense to take 1%, 5% and 10%
 - depends upon numbers of observations
- Important to save detail
 - P&L can be driven by very few actual positions

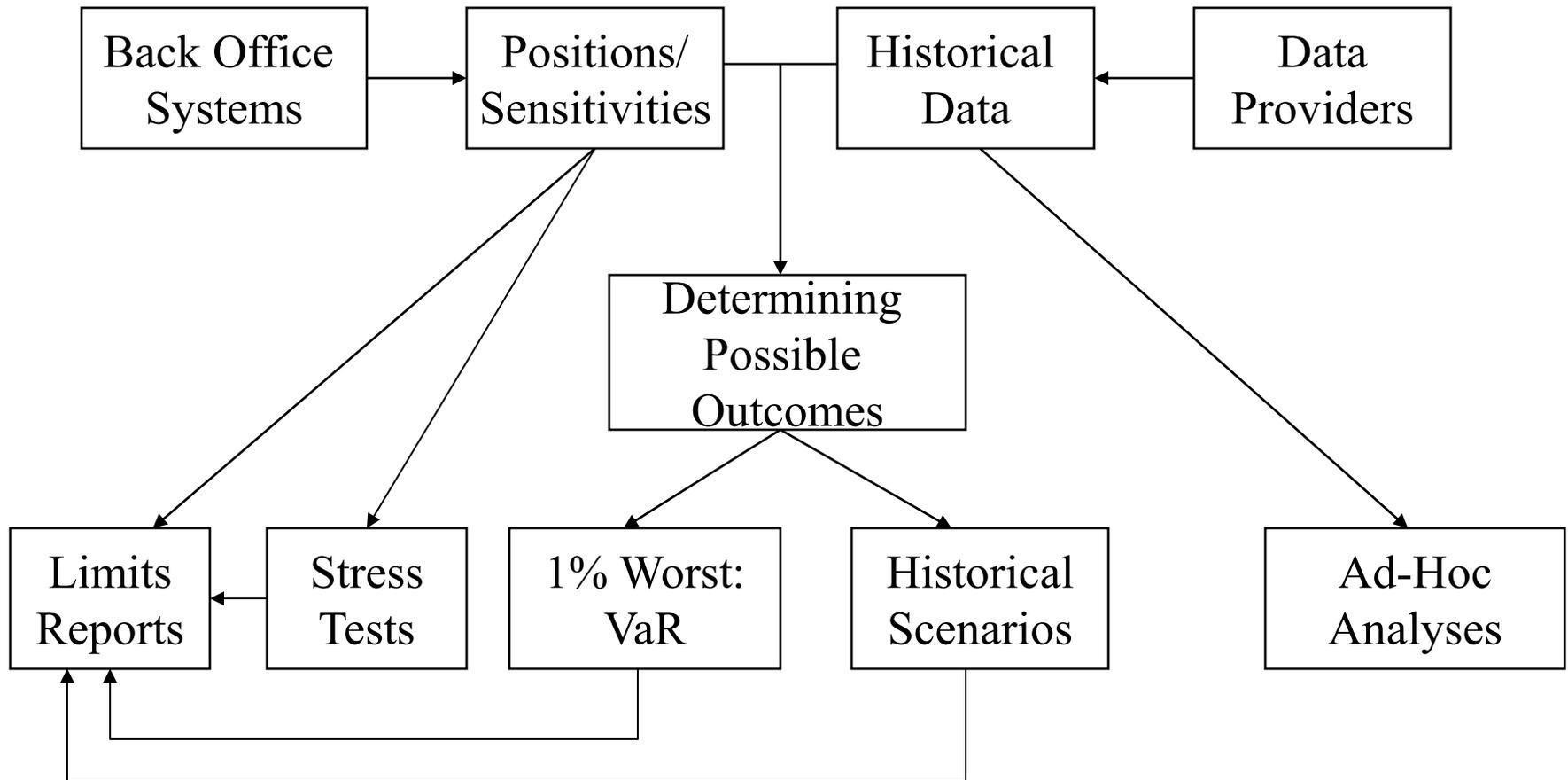
Now What?

- VaR numbers in isolation are worthless
- Need to calculate daily and compare
- Issues
 - use actual P&L or actual changes in indices
 - repricing vs. sensitivities
- Should be used in conjunction with Stress Testing and Scenario Analysis

Historical Simulation



Data Flow



Marginal VaR in Hist Sim

- VaR engine produces pnl *vectors* with 1043 pnl dates each (4 year history)
- For a given day, a parent pnl vector is sum of its child pnl vectors
e.g. Firm pnl = IED pnl + FID pnl
- If y is the parent (*eg. FIRM*) pnl vector and x_i are the children pnl vectors (*eg. IED and FID*)

$$y = \sum_{i=1}^n x_i$$

Variance and Covariance of Pnl Vectors

- Lets consider variance as a proxy for risks associated with pnl vectors

the variance of y is then

$$\sigma_y^2 = E[(y - \bar{y})^2] = E\left[\left(\sum_{i=1}^n (x_i - \bar{x}_i)\right)^2\right] = E\left[\sum_{j=1}^n \sum_{i=1}^n (x_i - \bar{x}_i)(x_j - \bar{x}_j)\right] = \sum_{j=1}^n \sum_{i=1}^n E[(x_i - \bar{x}_i)(x_j - \bar{x}_j)]$$

Noting that

$$E[(y - \bar{y})(x_j - \bar{x}_j)] = E\left[\sum_{i=1}^n (x_i - \bar{x}_i)(x_j - \bar{x}_j)\right] = \sum_{i=1}^n E[(x_i - \bar{x}_i)(x_j - \bar{x}_j)]$$

$$\sigma_y^2 = \sum_{j=1}^n E[(y - \bar{y})(x_j - \bar{x}_j)]$$

Define Marginal as Variance/Covariance Ratio

$$\sigma_y^2 = \sum_{j=1}^n E[(y - \bar{y})(x_j - \bar{x}_j)]$$

- Lets rewrite this expression as

$$\sigma_y^2 = \sum_{j=1}^n MVaR_j$$

where

$$MVaR_j = \frac{E[(y - \bar{y})(x_j - \bar{x}_j)]}{\sigma_y^2}$$

Marginal VaR using Variance/Covariance Ratio

- We can use the variance/covariance ratio derived before to distribute Parent level VaR into marginals for Children

$$\text{i.e. Marginal VaR for Child} = \frac{\text{Covariance (Parent , Child)}}{\text{Variance (Parent)}} * \text{Parent VaR}$$

$$\text{And Marginal VaR for FID} = \frac{\text{Covariance (Parent , FID)}}{\text{Variance (Parent)}} * \text{Parent VaR}$$

Hence, Parent VaR = Child Marginal VaR + FID Marginal VaR

Marginal VaR using Variance/Covariance Ratio

- Firm VaR and its marginals

Business Unit	VaR (All Runs)	Marg. VaR
	(81,411)	NA
	(14,481)	(2,822)
	(29,990)	(14,273)
	(72,457)	(64,316)
	(81,411)	<<Sum of marginals

- Marginals are relative to the immediate parent in the hierarchy.

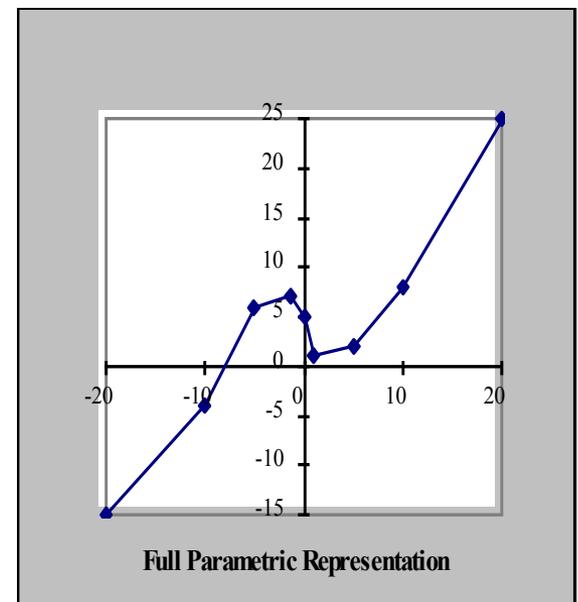
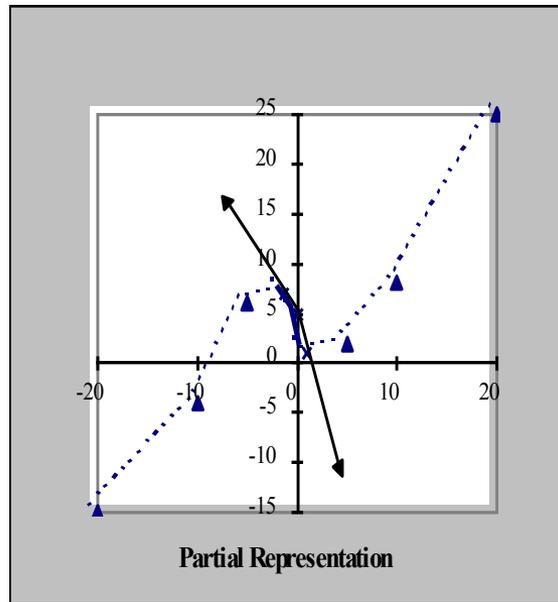
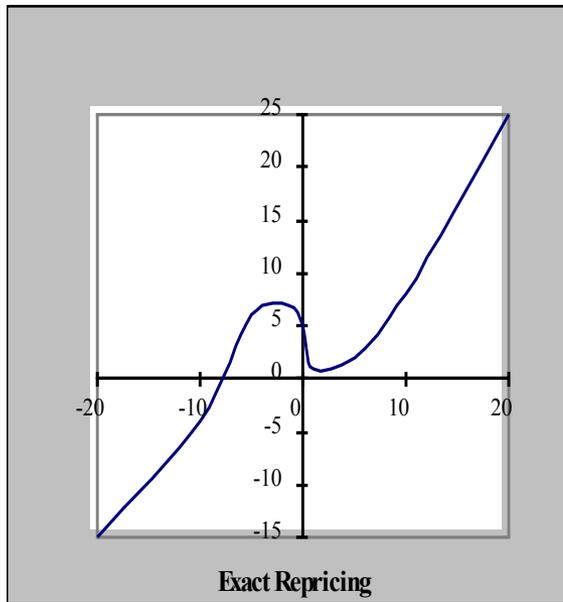
Business Unit	VaR (All Runs)	Marg. VaR	
	(81,411)	NA	
	(14,481)	(2,822)	Marginal to
	(29,990)	(14,273)	Marginal to
	(72,457)	(64,316)	Marginal to
	(26,387)	(12,118)	Marginal to
	(9,387)	(2,248)	Marginal to
	(1,139)	(212)	Marginal to
	(44,786)	(21,992)	Marginal to
	(50,208)	(32,450)	Marginal to
	(69,020)		--
			<i>Different from</i>

- Marginals for all children always adds up to it Parent Standalone VaR

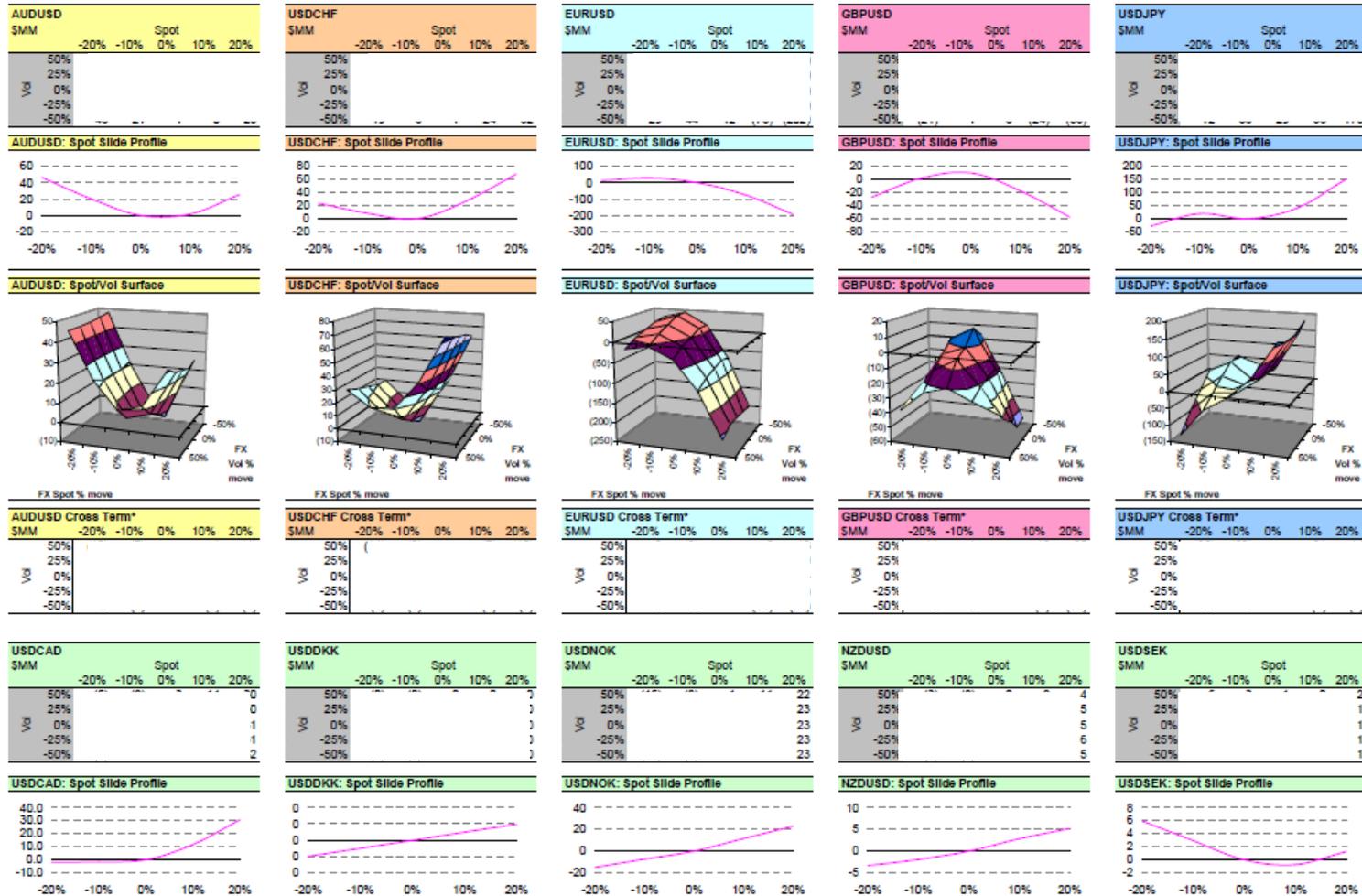
Advantages of an Empirical Approach

1. Intuitive appeal
2. Ease of error checking
3. Credit spread activity “built-in”
4. No maintained hypothesis of multivariate normality
5. Automatic scenario analysis

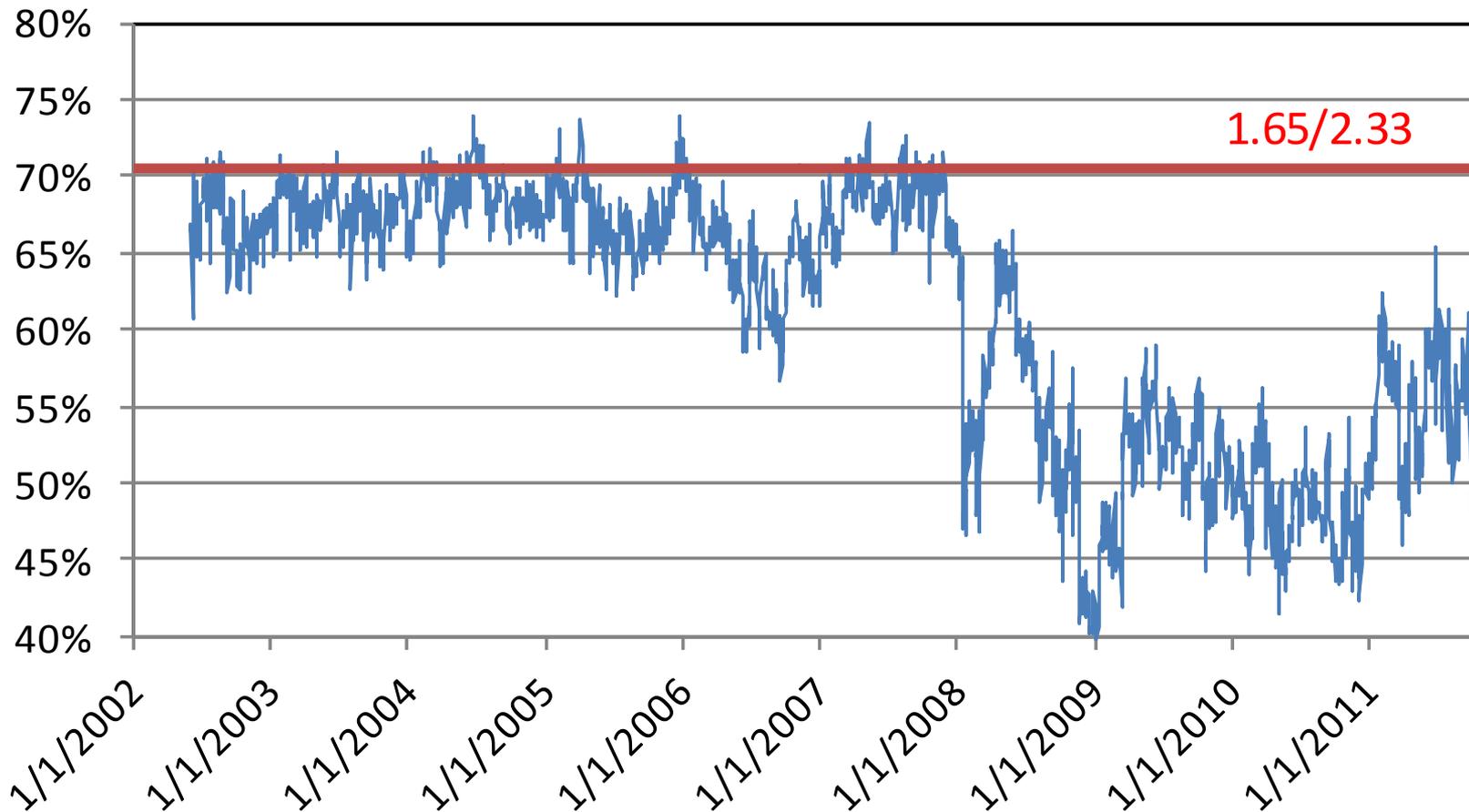
Using the Shocks for Repricing



Pricing Grids



95% - 99% VaR Ratio: 2001 - 2011



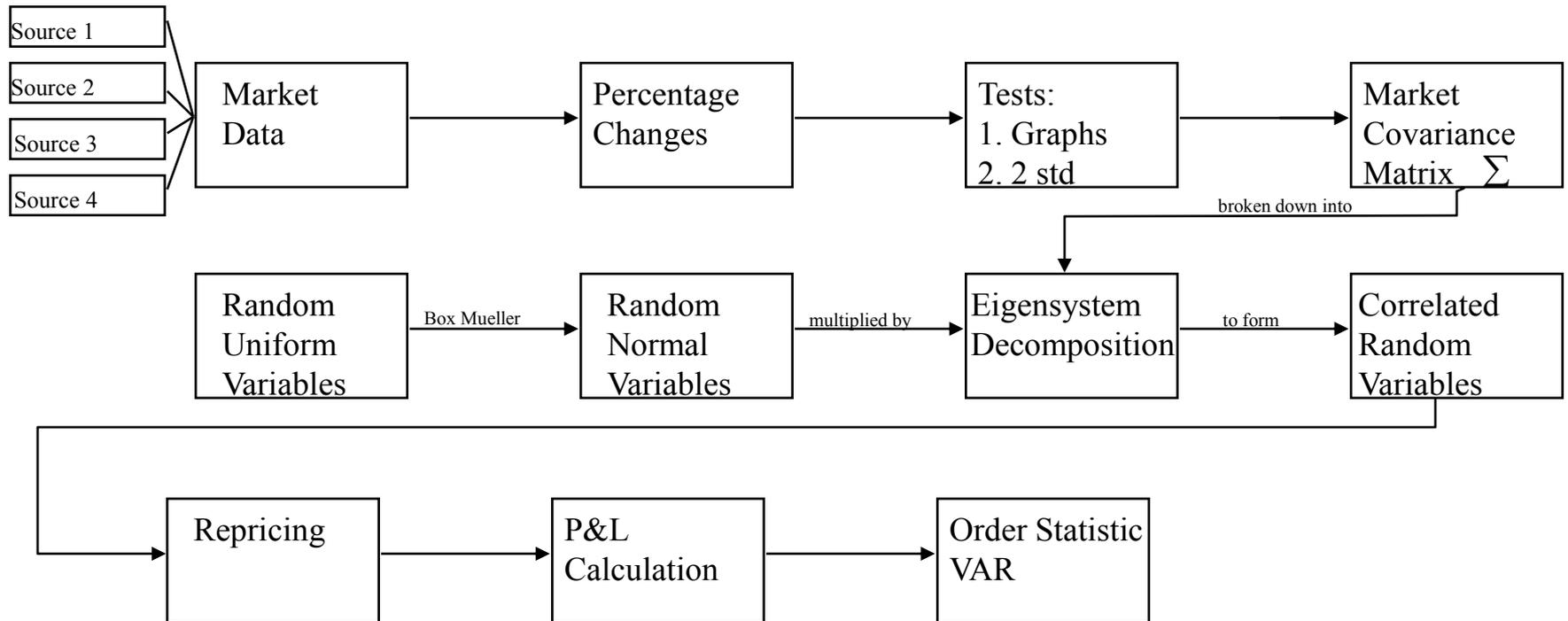
Description of Monte Carlo

1. Simulation method using sequences of random numbers to approximate values which cannot be determined analytically
2. Phrase coined in Manhattan project due to similarity to games of chance
3. Only requirement is that system be describable in terms of p.d.f.'s

Financial Applications

1. Path-dependent products
2. Convex portfolios for hedge analysis
3. Risk exposure for portfolios with strong “jump” risk (insurance sectors)
4. Credit exposure
5. VAR

The Monte Carlo Process



The Monte Carlo Process

1. Seven basic steps
 - a) Random uniform number generation
 - b) Random normal conversion
 - c) Covariance matrix generation
 - d) Covariance matrix factorization
 - e) Creation of correlated random “shock” variables
 - f) Use of shocks for repricing
 - g) Estimating the VAR

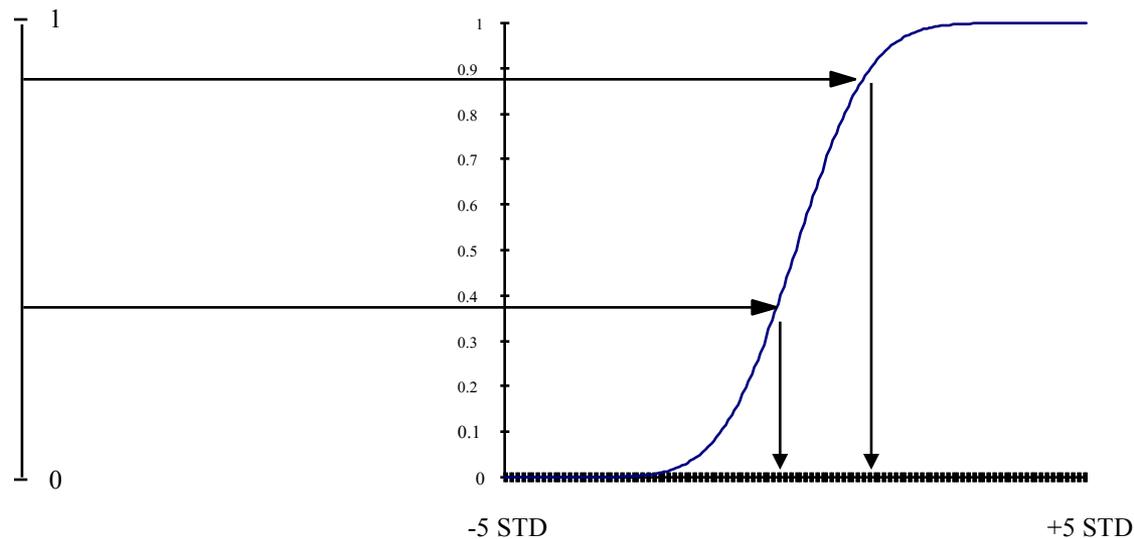
The Monte Carlo Process:

Random Uniform Number Generation

1. Three types of random numbers
 - a) truly random numbers
 - i. cannot be produced by computers
 - ii. must be from external source like radioactive decay
 - b) Quasi-random numbers
 - i. difficult to implement
 - ii. optimize uniformity
 - c) Pseudo-random numbers
 - i. generated by computer algorithm
 - ii. look independent and uniform
 - iii. prevailing method
 - iv. many algorithms available, see Knuth

The Monte Carlo Process: Conversion to Random Normal

1. Essentially, one needs to take a uniform 0,1 and convert to a normal 0,1 by mapping the the uniform draws to the CDF of the standard normal curve:



•see **Box-Mueller method in Numerical Recipes in C**

The Monte Carlo Process: Covariance Matrix Construction

1. Buy or Build
 - a) coverage
 - b) assumptions
 - i. missing data
 - ii. weighting
2. Differencing Interval
 - a) daily vs. weekly vs. monthly
 - b) scalability

The Monte Carlo Process: Matrix Factoring

1. Goal: to get a “square root” matrix
2. Computationally, the most complex part of Monte Carlo simulation
3. Details given in Numerical Recipes in C

The Monte Carlo Process: Matrix Factoring

- Two Methods
 - Cholesky
 - computationally simpler
 - used by several packages
 - requires positive definite matrix
 - create lower triangular such that $\Sigma = LL'$
 - Eigenvalue-Eigenvector decomposition
 - more complex
 - more robust
 - create E and Λ such that $\Sigma = E\Lambda E'$

The Monte Carlo Process: Creating Correlated Shocks

If R is an $n \times 1$ random normal vector $\Lambda^{1/2}$ is an $n \times n$ diagonal matrix containing the square roots of the Eigenvalues and E is a matrix of the Eigenvectors, then

$$R\Lambda^{1/2} E' \sim N(0, \Sigma)$$

The Monte Carlo Process: Creating Correlated Shocks

- Checking the process
 - It is difficult to make small mistakes
 - Ways to check the factoring and the random correlated variables
 - re-creation of covariance matrix
 - test the variances with F or chi-squared tests
 - $\sigma_o^2 / \sigma^2 \sim F(m-1, k-1)$
 - test the correlations with z-tests
 - $\rho \sim N(\rho_o, (1-\rho_o^2)/n)$
 - Box's M test

The Monte Carlo Process: Creating Correlated Shocks

- Potential Problems
 - precision
 - negative eigenvalues
 - computation time
 - data storage
 - error audit trail

Weaknesses

- Assumption of multivariate normality
 - serial correlation
 - skewness
 - kurtosis
- Must adjust to simulate mean reversion
- Must adjust to simulate jump diffusion

Using the Shocks for Repricing

- Now what?
- Two basic approaches
 - Exact repricing
 - More exact
 - computationally burdensome
 - Parametric representation
 - can be very accurate
 - cross-partial derivatives can create some noise

Estimating the VAR

- Once you reprice, calculate P&L
- Order P&L results
- Take nth largest case to get VAR
- Need to group and sort to get subportfolio VAR

Backtesting

- What is a backtest?
 - Comparison of VaR vs. P&L
 - Usually VaR @ time t and P&L @ $t+1$
- Why backtest?
 - You are required to
 - Provides a reality check on your calculations
 - Helps find errors
 - Identifies changes in risk profile

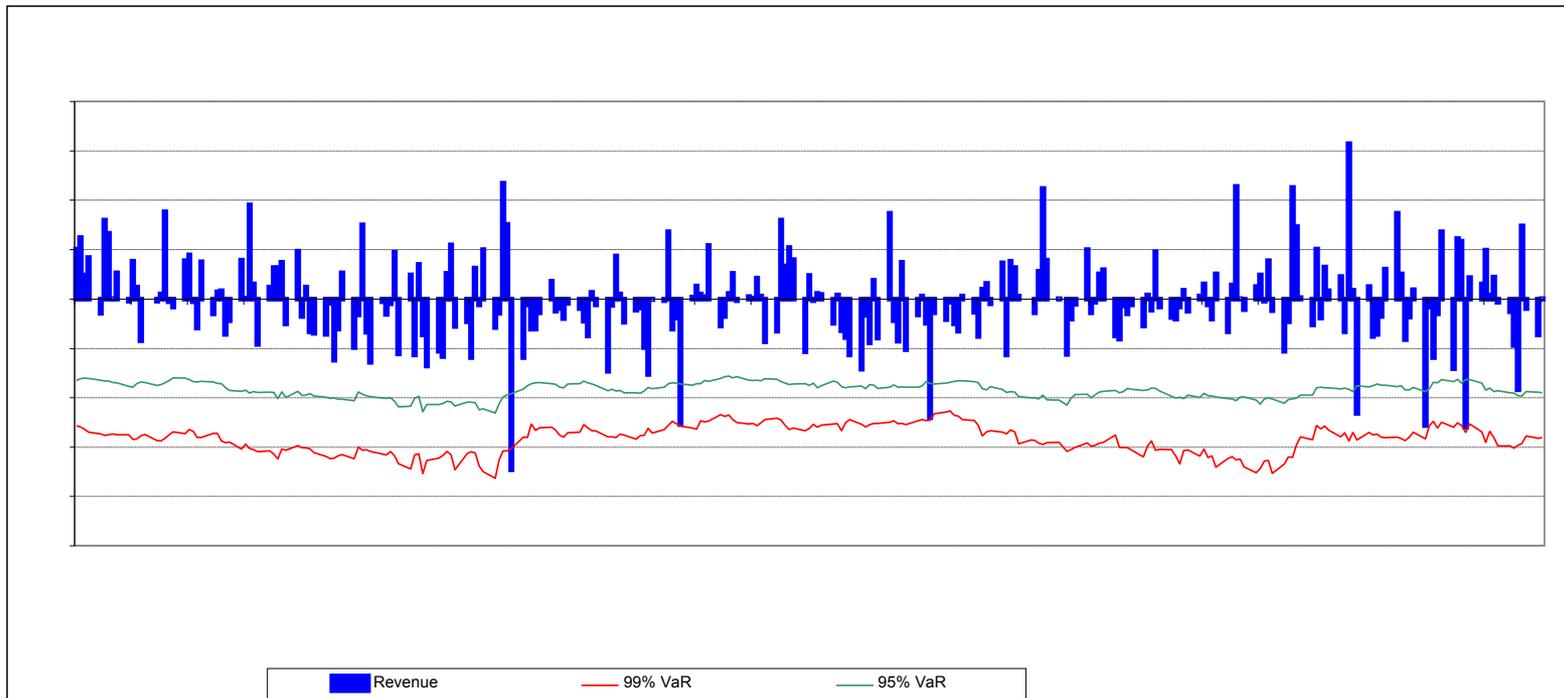
Backtests

- Granularity
 - Down to desk level
 - (Need to document outliers)
 - Often you need to aggregate
 - Aggregation hides spikes
 - Split books a problem

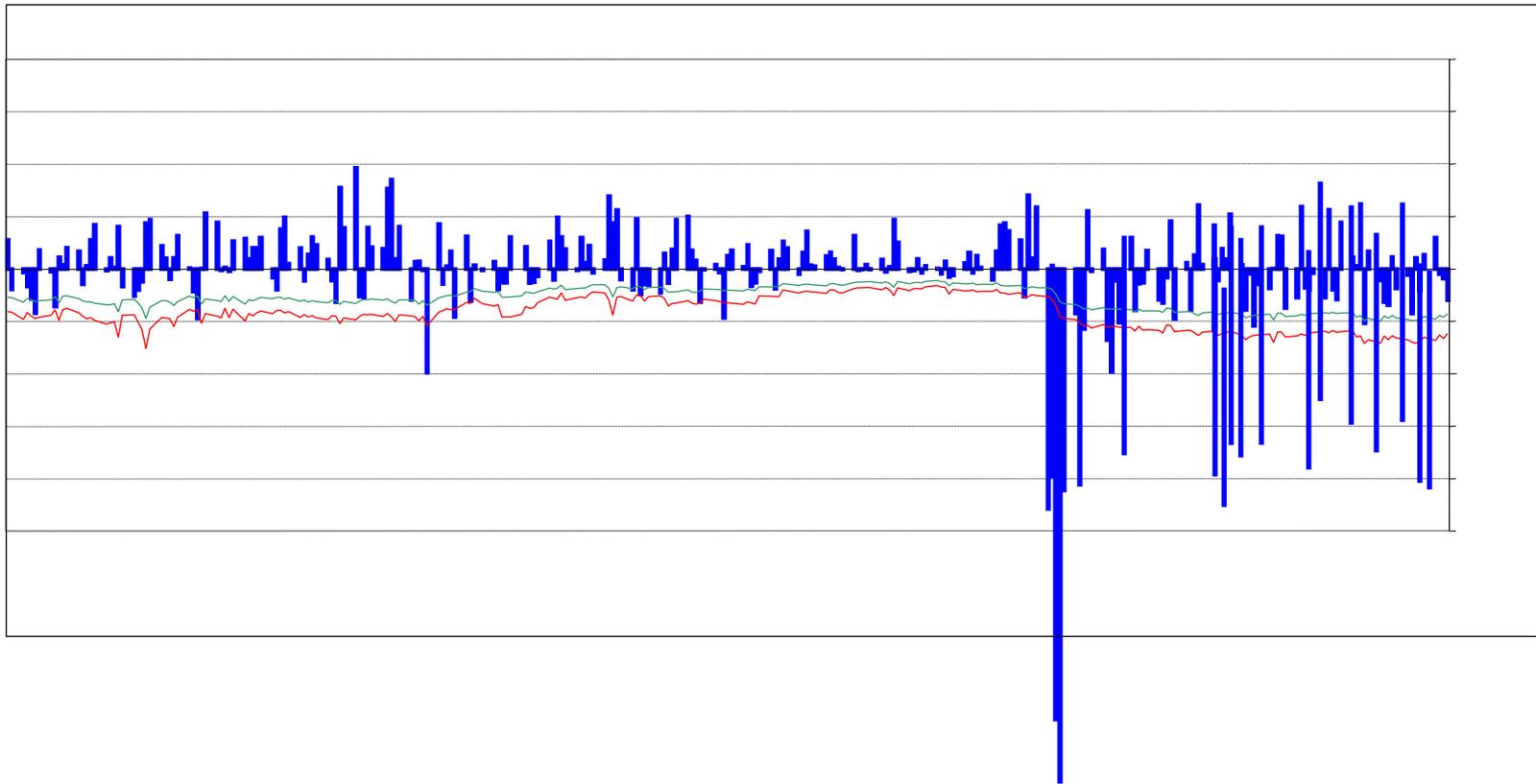
Backtests

- Issues
 - Split books
 - Front office or back office P&L
 - Timing of P&L recognition
 - Re-orgs
 - Data maintenance
 - Frequency of updates
 - Include upside jumps?

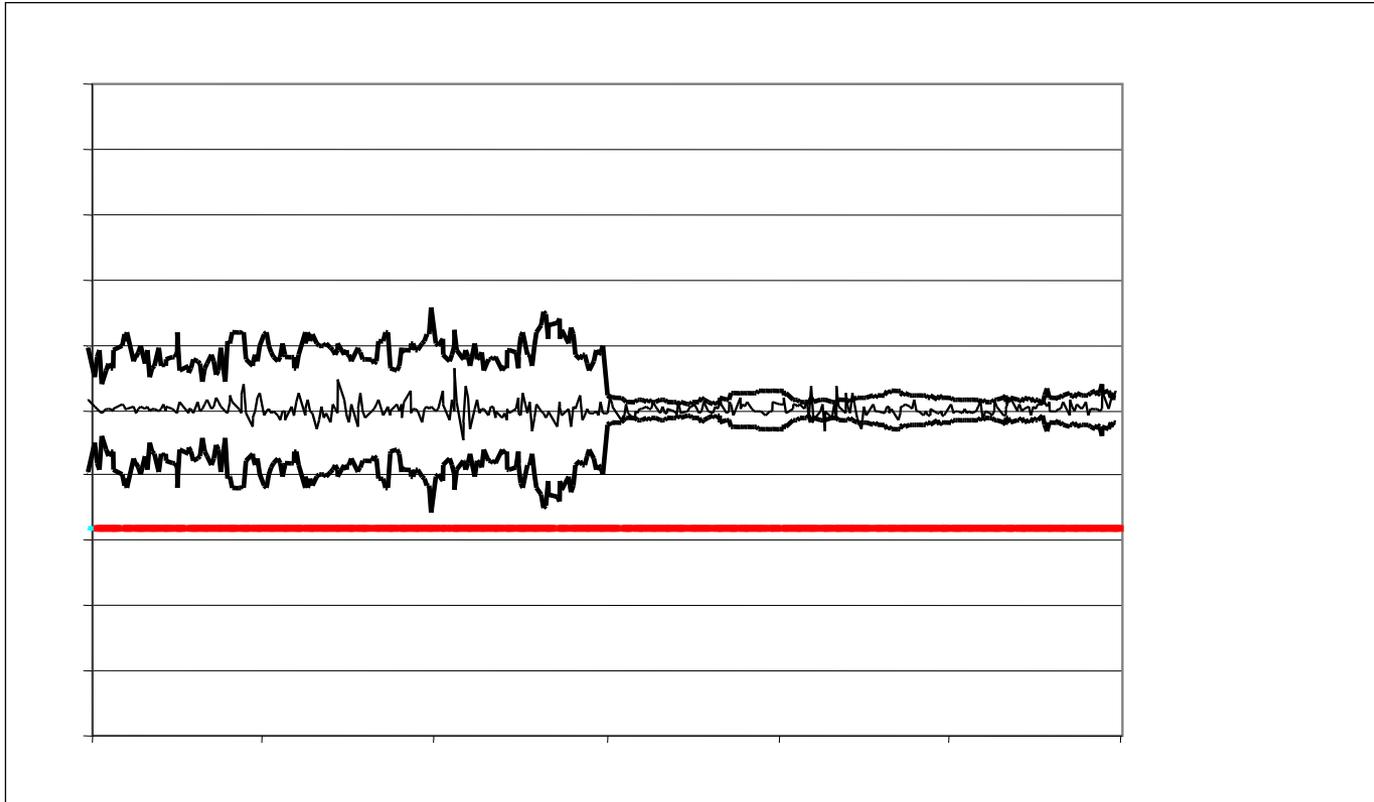
Backtesting



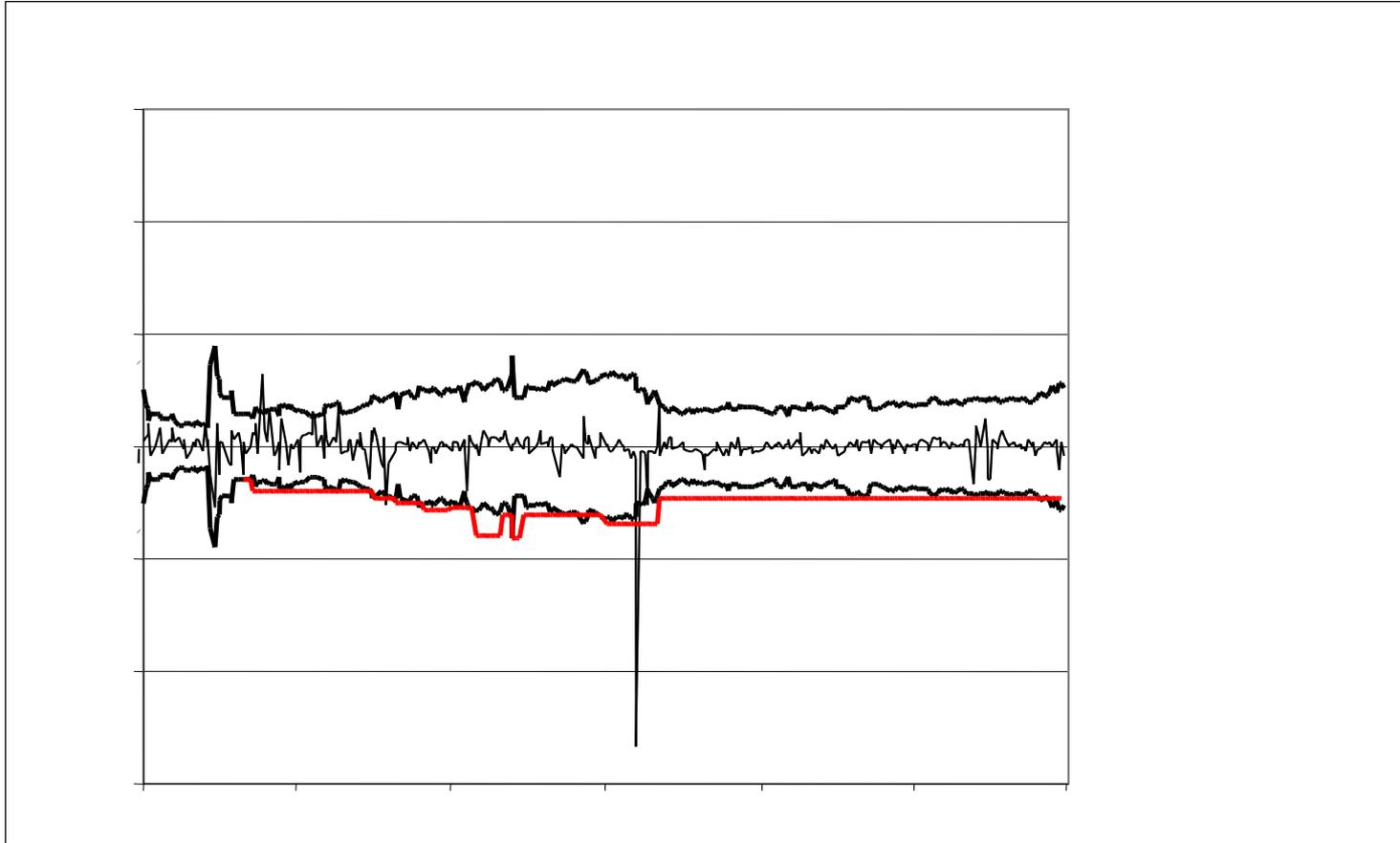
Backtests



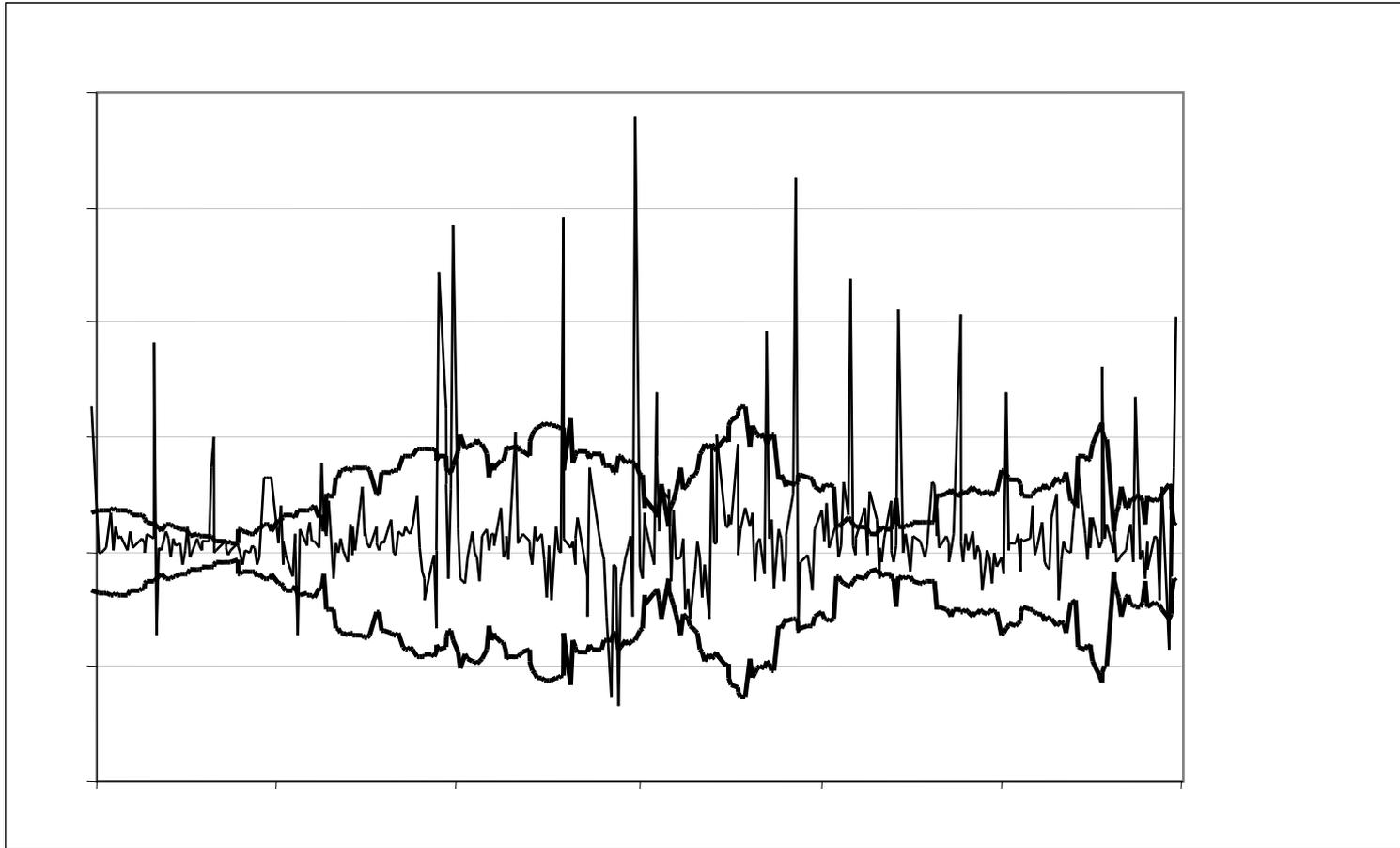
Examples



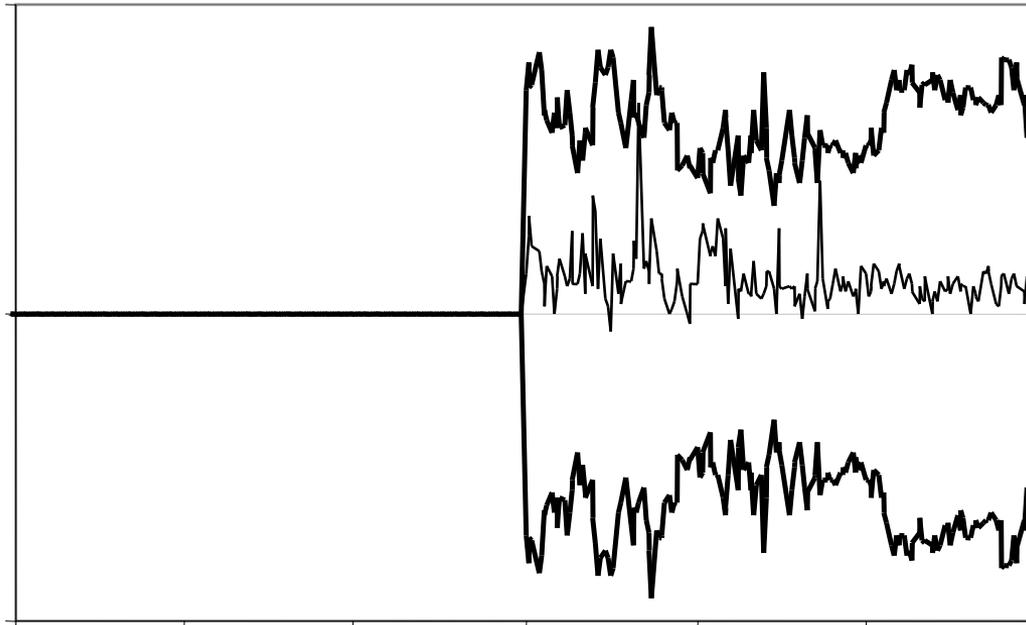
Examples



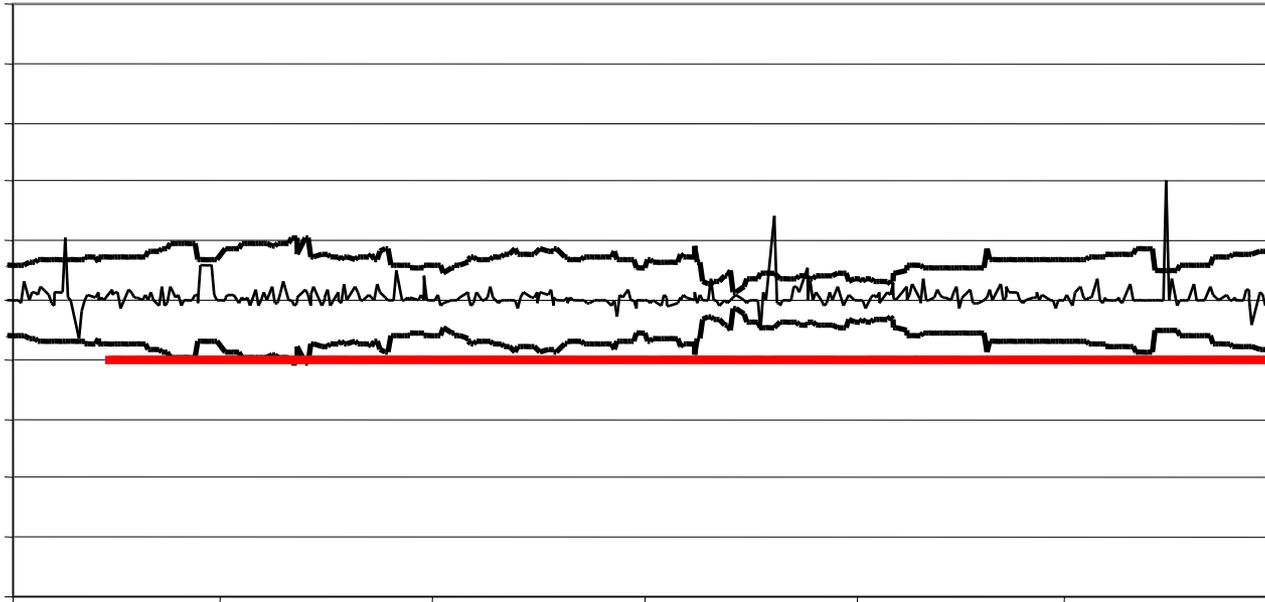
Examples



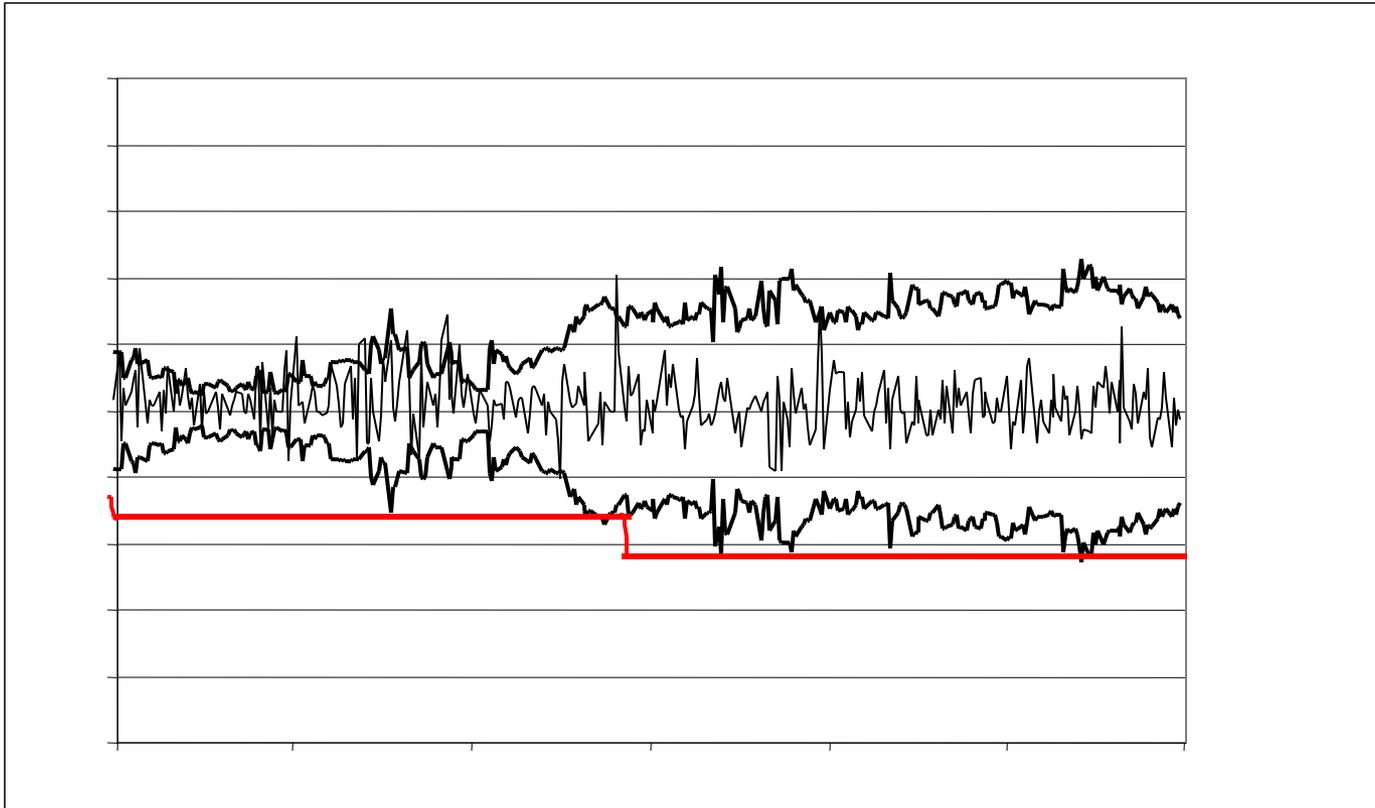
Examples



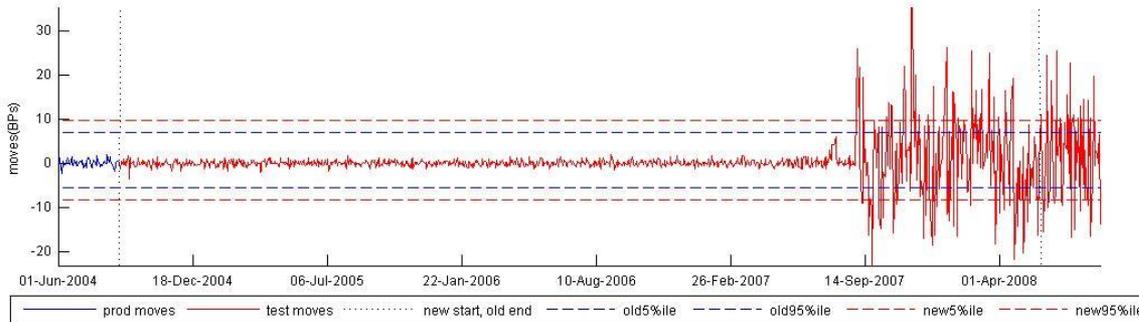
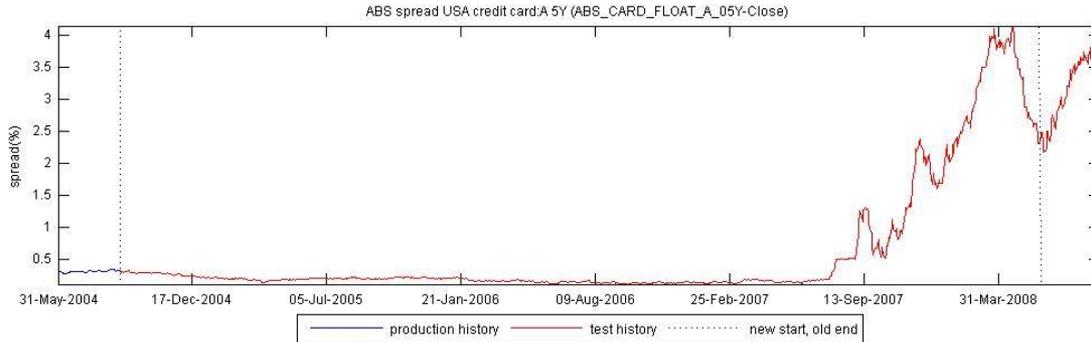
Examples



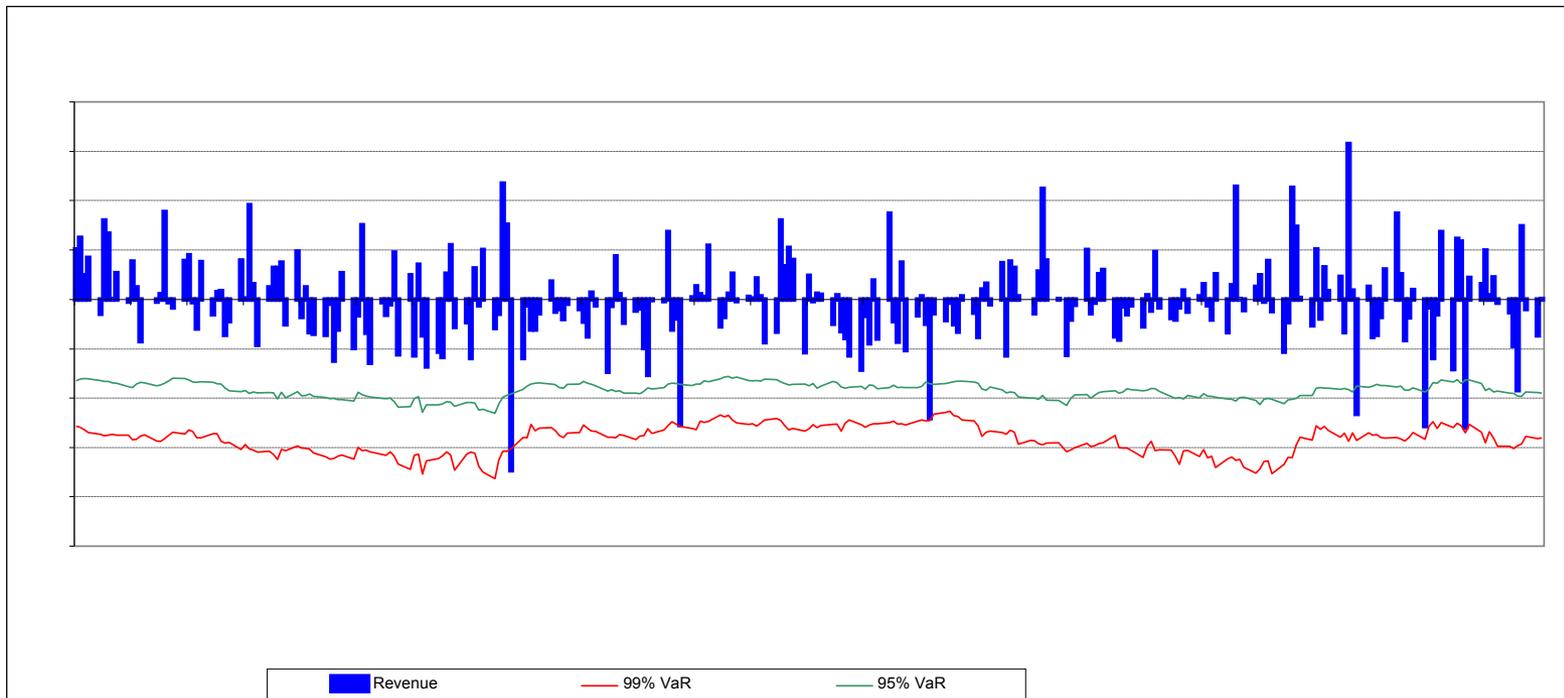
Examples



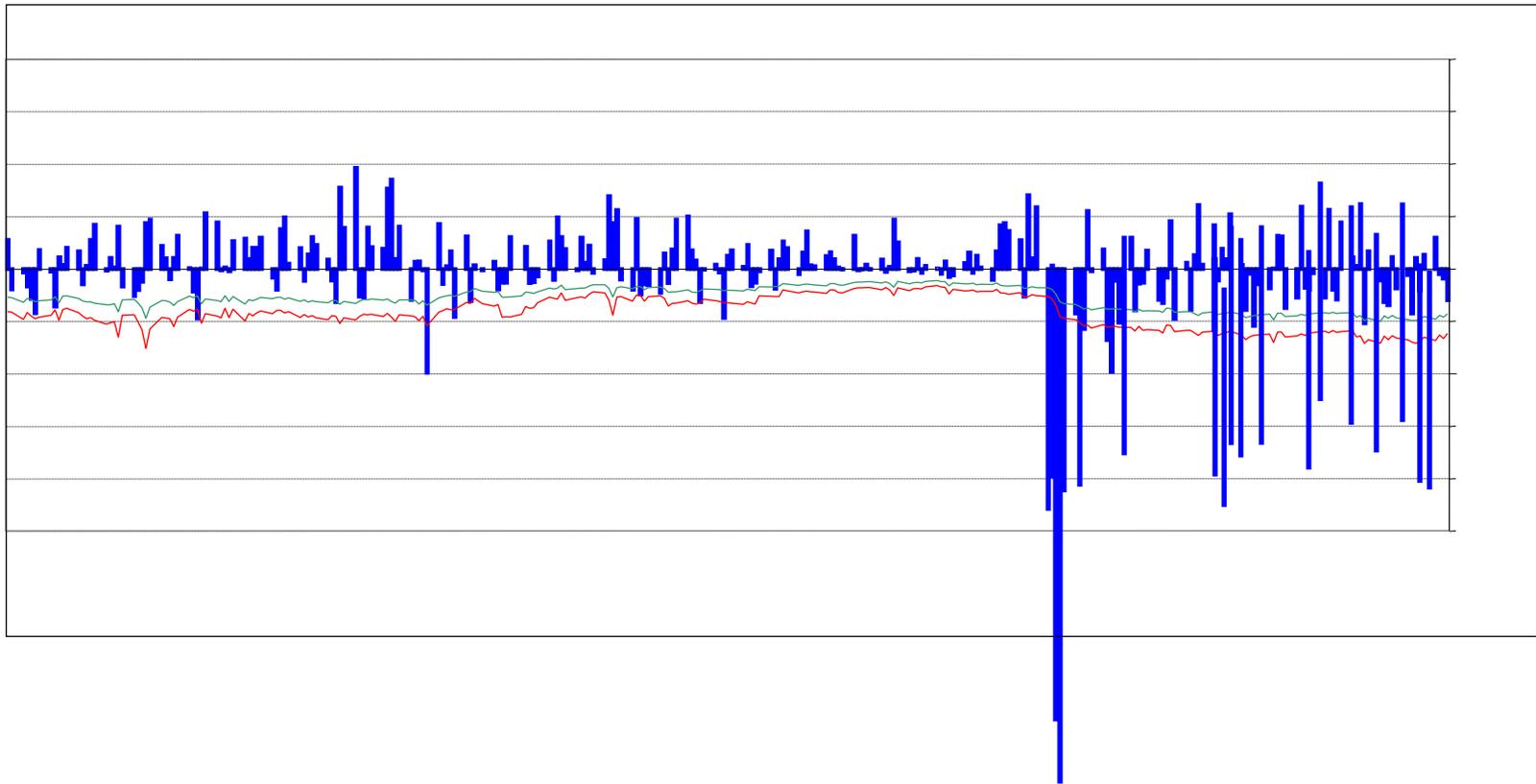
Data



Backtesting



Backtests



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