# Lecture 26: Introduction to Counterparty Credit Risk

## - Enterprise-Level Derivatives Modeling


Yi Tang, Managing Director,
Head of Cross Asset Modeling Group, Morgan Stanley


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# Overview of Counterparty Credit Risk

- In OTC (Over The Counter) derivatives
  - Counterparty (CP) credit risk - Our counterparty (CP) will not pay us the full amount it owes us if it defaults (bankruptcy, failure to pay, …)
    - Default risk
    - MTM risk due to the likelihood of CP future default, CP credit spread widening
- Similar to (corporate) bonds (in terms of economics)
  - Except that the credit risk in bonds is issuer risk

- Credit Valuation Adjustment (CVA)
  - Price of counterparty (CP) credit risk, mainly MTM risk due to the likelihood of CP future default
  - An adjustment to the price/MTM from a CP-default-free model/broker quote
  - Typically no need of CVA for bonds (and some other products)
  - Also a part of the Basel 3 Credit Capital (CVA add-on)
  - In general, cannot be priced with trade-level derivatives models
    - Prevailing derivatives models
    - Due to non-linear portfolio effect
  - One of the applications of enterprise-level derivatives models
    - Model non-linear risks/effects/metrics in a derivatives portfolio

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# Examples and Questions

- You have an OTC derivatives trade (e.g., an IR swap) (or a portfolio of trades)
  - With no collateral
  - We know nothing about counterparty credit risk (or credit valuation adjustment, CVA)

- Trade PV was zero on day 1 (excluding CVA)
- Trade PV became $100MM later on (excluding CVA)
- Then, your counterparty (unexpectedly) defaults with 50% recovery
  - You get paid $50MM cash (= $100MM x 50%)

- Have you made $50MM or lost $50MM over the life of the trade in your trading operation, or are you flat?

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# CVA (Credit Valuation Adjustment)

- This is where CVA (Credit Valuation Adjustment) comes in
  - Price of counterparty credit risk
  - Make whole on counterparty default loss
    - If hedged with CVA Desk (by buying credit protection on CP)

- CVA (back of the envelope) approximation
  - CVA (on receivables, charge to counterparty):
    - MPE * CP CDS Par Spread * Duration
  - MPE: Mean Positive Exposure

  - More accurate formula for MPE CVA

$$CVA_{MPE} = -E_0^Q \left[ 1_{\{\tau \le T\}} \frac{\left(V_{\tau-} - C_{\tau-}\right)^+ \left(1 - R_\tau\right)}{\beta_\tau} \right]$$

$T$: Final maturity of CP portfolio
$\tau$: The CP (first) default time
$V_{\tau-}$: CP portfolio value, $C_{\tau-}$: CP posted collateral, both as of time $\tau-$
$R_\tau$: CP recovery as of time $\tau$

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# CVA (Credit Valuation Adjustment)

- Nonlinear portfolio effects in CVA (requiring enterprise-level modeling)
  - Offsetting trades

  - Asymmetry of one's net receivables (or assets) and one's net payables (or liabilities) given counterparty default
    - Option-like payoff

  - A counterparty (in one netting group) typically trades many derivatives instruments cross assets (such as IR, FX, credit, equity, commodities, and mortgages)
    - Option on a basket of (cross asset) derivatives trades

  - Ideally, all trade-level models (and martingale targets) should be accessible or called by the enterprise-level model in run-time, among others

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# CVA (Credit Valuation Adjustment)

- (Potential refinements needed)

- Asset/MPE CVA (for receivables w.r.t. CP default) (cost)

$$CVA_{MPE} = -E_0^Q \left[ 1_{\{\tau \le T\}} \frac{(V_{\tau-} - C_{\tau-})^+ (1 - R_\tau)}{\beta_\tau} \right]$$

- Liability/MNE CVA (for payables w.r.t. self default) (benefit) (Model scenario)

$$CVA_{MNE} = -E_0^Q \left[ 1_{\{\bar{\tau} \le T\}} \frac{(V_{\bar{\tau}-} - C_{\bar{\tau}-})^- (1 - \bar{R}_{\bar{\tau}})}{\beta_{\bar{\tau}}} \right]$$

- MPE/MNE: Mean Positive/Negative Exposure

- First to default?

# Examples and Questions

- You have a derivatives trade (e.g., an IR swap) (or a portfolio of trades)
  - With no collateral

- Trade PV was zero on day 1
- Trade PV became +$100MM later on
- The market risk and counterparty credit risk are properly hedged
- Do you have any other risks?

- (Cashflow) Liquidity/funding risk
  - Need funding for uncollateralized derivatives receivables
  - Cash outflow in futures or collateralized hedges
  - Contingent funding risk
  - Funding benefit from uncollateralized derivatives payables

- Other risks
  - Unexpected/tail risks, (equity) capital

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# Examples and Questions

- Selling put options or put spreads
- Selling put options on a stock versus buying the stock outright
  - On stocks with or without dividend?
- Warren Buffett/Berkshire sold long dated puts on four leading stock indices in U.S., U.K., Europe, and Japan
  - Collated about $4Bn premium without posting collateral
  - Page 16 in http://www.berkshirehathaway.com/letters/2012ltr.pdf
  - Stock indices:
    - S&P 500, FTSE 100, Euro Stoxx 50, Nikkei 225
  - Non-linear portfolio risks to dealers (requiring enterprise-level modeling)
    - CVA, wrong-way risk
    - Liquidity/funding, wrong-way risk
    - Risk management of CVA, Liquidity/funding - Laying off such risks
      - Credit-linked note, intermediation
      - Tranched portfolio credit protection

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# Examples and Questions

- From the perspective of Warren Buffett/Berkshire
  - Selling insurance on the equity market
  - Day one cash inflow (low cost funding)
  - No cash outflow until trade maturity
  - Warren Buffett/Berkshire explored the feasibility of posting collateral

- Who is making (or will make) money and who is losing (or will lose) money?
  - Where does the value come from?

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# CVA Conundrum

- You trade with counterparty A
- You buy credit protection on A from counterparty B
- You buy credit protection on B from counterparty C
- You buy credit protection on C from counterparty D
- . . .

- It becomes an infinite series

- What is the impact on CVA pricing?
  – Arbitrage pricing in terms of replication using hedge instruments

- What strategies can you apply to handle this efficiently in practice?
  – You buy credit protection on A from counterparty B fully collateralized

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# Overview of Enterprise-Level Derivatives Modeling

- Trade-level derivatives models
  - Model each trade independently (PV and Greeks)
  - The PV and Greeks of a portfolio is the linear aggregation of those of each trade in the portfolio
  - Focus of prevailing derivatives modeling
  - More modeling efforts needed to understand the complete picture of the economics and risks of OTC derivatives business

- Enterprise-level derivatives models
  - Model non-linear risks/effects/metrics in a portfolio
    - Such risks/effects/metrics of a portfolio are NOT a linear aggregation of those of each trade in the portfolio
    - CVA, funding, RWA/capital, liquidity are examples.
    - Model all trades and market/risk factors of a portfolio consistently (with proper joint distributions of the underlying market/risk factors)
      - Leveraging the trade-level models
    - Feedback to trade-level models – Cancellable swap facing a CP close to default
  - Significant requirements in modeling, infrastructure, and data
    - Martingale testing, remsampling, interpolation, and modeling
    - Not studied enough in terms of systematic approaches

# Examples of Martingales and Martingale Measures

- Martingale measures for
  - Forward price
  - Forward LIBOR and swap rate
  - Forward FX rate
  - Forward CDS par coupon
  - Do not consider collateral discounting explicitly

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# Examples of Martingales and Martingale Measures

- Risk neutral measure Q
  - Numeraire $\beta(t)$
  - $Y_t/\beta(t)$ is a Q-martingale, $Y_t$ is the price of a traded asset with no intermediate cashflows (Harrison and Pliska martingale no-arbitrage)

$$\beta(t) = \exp\left(\int_0^t r(u)\,du\right) \qquad Y_s/\beta(s) = E_s^{\mathbf{Q}}\left[Y_t/\beta(t)\right] \qquad t \geq s \geq 0$$

- Forward arbitrage-free measure (as of $T$) $P_T$
  - Numeraire $P(t,T)$
  - $Y_t/P(t,T)$ is a $P_T$-martingale,

$$Y_s/P(s,T) = E_s^T\left[Y_t/P(t,T)\right] \qquad T \geq t \geq s \geq 0$$

$$Y_s = P(s,T)E_s^T\left[Y_T\right] \qquad T = t \geq s \geq 0$$

- Forward price $F_Y(t,T) = Y_t/P(t,T)$ is a $P_T$-martingale.

- Harrison, J. M. and S. R. Pliska (1981), "Martingales and Stochastic Integrals in the Theory of Continuous Trading," Stochastic Processes and their Applications, 11, 215-260.

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# Examples of Martingales and Martingale Measures

– Forward LIBOR $L_i(t)$

$$1+\delta_i L_i(t)=P(t,T_i)/P(t,T_{i+1}) \quad (t\le T_i) \quad \Rightarrow L_i(t) \text{ is a } \mathbf{P}_{T_{i+1}}\text{-martingale.}$$

$$L_i(s)=E_s^{T_{i+1}}\left[L_i(t)\right] \quad \left(0\le s\le t\le T_i\right)$$

– Annuity arbitrage-free measure

$$\mathbf{P}_{A_{i,N}}\text{: Numeraire: } A_{i,N}(t)=\sum_{j=i+1}^{N}\delta_{j-1}^{S}P\left(t,T_j^S\right)$$

$$Y(s)/A_{i,N}(s)=E_s^{A_{N,i}}\left[Y(t)/A_{i,N}(t)\right] \quad \left(0\le s\le t\le T_j^S\right)$$

– Forward swap rate (for accrual period from $T_i^S$ to $T_N^S$)

$$S_{i,N}(t)=\left(P\left(t,T_i^S\right)-P\left(t,T_N^S\right)\right)/A_{i,N}(t) \quad \left(t\le T_i^S\right)$$

$$S_{i,N}(t) \text{ is a } \mathbf{P}_{A_{i,N}}\text{-martingale} \quad \left(t\le T_i^S\right)$$

$$S_{i,N}(s)=E_s^{A_{N,i}}\left[S_{i,N}(t)\right] \quad \left(0\le s\le t\le T_j^S\right)$$

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# Examples of Martingales and Martingale Measures

- Let $S_{FX}(t)$ denote the time t spot FX exchange rate in terms of the value or price of one unit of foreign currency in terms of domestic currency.
- Furthermore, let $P^D(t,T)$ and $P^F(t,T)$ denote the time t price of a domestic and foreign (default-free) zero-coupon bond paying one unit of domestic and foreign currency with maturity T, respectively.

- FX forward rate

$$F_{FX}(t,T) \equiv \frac{S_{FX}(t)P^F(t,T)}{P^D(t,T)} \qquad (0 \le t \le T)$$

$$F_{FX}(s,T) = E_s^{T,D}\left[F_{FX}(t,T)\right] \qquad (0 \le s \le t \le T)$$

- Domestic forward measure $\mathbf{P}_{T,D}$ with numeraire of $P^D(t,T)$

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# Change of Probability Measure

- Typically, the model is solved in one measure, the martingale representations are under various martingale measures. Thus, changes of measures are needed.

- Change of measure = change of numeraire.

- Let $X_t$, $N_1(t)$>0, $N_2(t)$>0 be the arbitrage prices of simple traded assets (with at most one cash flow at maturity).

- $X_t$ is measure invariant, and

$X_t / N_1(t)$ is a $\mathbf{P}_{N_1}$-martingale and $X_t / N_2(t)$ is a $\mathbf{P}_{N_2}$-martingale, thus

$$X_t = N_1(t) E_t^{N_1}\left[X_T / N_1(T)\right] = N_2(t) E_t^{N_2}\left[X_T / N_2(T)\right] \qquad (t \le T)$$

- Let $Y_T = X_T/N_1(T)$, then

$$E_t^{N_1}\left[Y_T\right] = E_t^{N_2}\left[Y_T N_1(T)/N_2(T)\right] / \left(N_1(t)/N_2(t)\right) \qquad (t \le T)$$

- Particularly,

$$\begin{aligned} E_t^{T_i}\left[Y_T\right] &= E_t^{T_{i+1}}\left[Y_T P(T,T_i)/P(T,T_{i+1})\right] / \left(P(t,T_i)/P(t,T_{i+1})\right) \\ &= E_t^{T_{i+1}}\left[Y_T\left(1+\delta_i L_i(T)\right)\right] / \left(1+\delta_i L_i(t)\right) \qquad (t \le T \le T_i) \end{aligned}$$



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# Martingales and Martingale Measures for Credit Derivatives

- What about credit derivatives?
- Survival probability measure and credit spread market model
  - Schönbucher, P. J. (2003), "A Note on Survival Measures and the Pricing of Options on Credit Default Swaps," downloadable from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.14.1824&rep=rep1&type=pdf
- Since $1_{\{t<\tau_j\}}\tilde{S}_j\left(t,T_i,T_{i+1}\right)\tilde{A}_j\left(t,T_i,T_{i+1}\right)$ is the time $t$ value of the premium leg of the CDS for the $j^{th}$ credit name where $\tau_j$ is the default time of the $j^{th}$ credit name) which is a traded asset, thus we have the following martingale relationship

$$\frac{1_{\{t<\tau_j\}}\tilde{S}_j\left(t,T_i,T_{i+1}\right)\tilde{A}_j\left(t,T_i,T_{i+1}\right)}{\beta(t)} = E_t^{\mathbf{Q}}\left[\frac{1_{\{T<\tau_j\}}\tilde{S}_j\left(T,T_i,T_{i+1}\right)\tilde{A}_j\left(T,T_i,T_{i+1}\right)}{\beta(T)}\right]$$

$$\left(0\le t\le T\le T_i<T_{i+1}\right)$$

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# Martingales and Martingale Measures for Credit Derivatives

- What is the martingale measure for CDS par coupon $\tilde{S}_j\left(t, T_i, T_{i+1}\right)$ ?
  - Starting point of martingale modeling factory
  - Next steps are martingale representation and change of probability measure

- Survival annuity probability measure (Schönbucher, 2003)

$$\left.\frac{d\mathbf{P}_{\tilde{A}_j}}{d\mathbf{Q}}\right|_t \equiv \frac{1_{\{t<\tau\}}\tilde{A}_j\left(t, T_i, T_{i+1}\right)}{\beta(t)\tilde{A}_j\left(0, T_i, T_{i+1}\right)}$$

$$\left(0 \le t\right)$$

$$\tilde{S}_j\left(t, T_i, T_{i+1}\right) = E_t^{\tilde{A}_j}\left[\tilde{S}_j\left(T, T_i, T_{i+1}\right)\right]$$

$$\left(0 \le t \le T \le T_i < T_{i+1}\right)$$

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# Martingale Testing, Remsampling, and Interpolation

- Beneficial for both trade-level and enterprise-level models
- Martingale testing: Testing against known martingale relationships
- Martingale remsampling: (Linear, log-linear) Transformation of simulated variables so that they exactly match given martingale relationships in numerical implementation (while keeping given variance metric unchanged)
  - Quadratic resampling

$$X = \frac{Std(X)}{Std(X_0)}\left(X_0 - E\left[X_0\right]\right) + E\left[X\right]$$

- Martingale testing interpolation: Guarantees the martingale relationships on the interpolated variables

$$M\left(s,T\right) = E_s\left\lfloor M\left(t,T\right)\right\rfloor \qquad \left(s \le t\right)$$

$$M\left(t,T_3\right) = \frac{M\left(s,T_3\right) - M\left(s,T_2\right)}{M\left(s,T_1\right) - M\left(s,T_2\right)} M\left(t,T_1\right) + \frac{M\left(s,T_1\right) - M\left(s,T_3\right)}{M\left(s,T_1\right) - M\left(s,T_2\right)} M\left(t,T_2\right)$$

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# Example of Martingale Modelling

- The LIBOR Market Model

$$dL_i(t)/L_i(t) = \lambda_i(t) dW_t^{T_{i+1},L_i} \qquad (0 \le t \le T_i)$$

where $W_t^{T_{i+1},L_i}$ is a 1-dimensional $P_{T_{i+1}}$-Brownian motion for $L_i(t)$.

- Change to the same measure for numerical implementation

$$\frac{dL_i(t)}{L_i(t)} = -\sum_{j=i+1}^{N-1} \frac{\lambda_i \lambda_j \delta_j L_j(t)}{1+\delta_j L_j(t)} \mathrm{Corr}\left(d\ln(L_i(t)), d\ln(L_j(t))\right) dt + \lambda_i dW_t^{T_N,L_i}$$

$$(0 \le t \le T_i)$$

where $W_t^{T_N,L_i}$ is a 1-dimensional $P_{T_N}$-Brownian motion for $L_i(t)$ satisfying

$$\mathrm{Corr}\left(dW_t^{T_N,L_i}, dW_t^{T_N,L_j}\right) = \mathrm{Corr}\left(d\ln(L_i(t)), d\ln(L_j(t))\right)$$

- This is the LIBOR market model in a general form (full dimensional model) with as many factors as there are number of LIBORs to be modelled. The correlations among all LIBORs are inputs to the model.

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# Examples and Questions

- Normally, we (dealer/investment bank) need to charge our counterparty CVA due to its likelihood of default.
- How to structure a trade whereby we can actually pay our counterparty due to its likelihood of default?
- Hint: The trade has a positive PV to us and is roughly an increasing function of the counterparty default probability

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# Examples and Questions

- Counterparty gives up benefit upon its default

- Extinguisher or zero-recovery swap
  - Knocks out at zero value if counterparty or either party defaults

- We have more future payables to our counterparty than receivables

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# Examples and Questions

- Extinguisher on a liability management swap of an emerging market sovereign (or other) entity for a bond issuance denominated in foreign/stronger currency

- The emerging market sovereign entity issues a bond denominated in USD, but would like to fund with its local currency. So it enters a cross currency swap with a dealer/investment bank to receive USD and pay its local currency.
- The dealer has expected future payables on such swap.
- Due to FX forward (or interest rate differential), FX and credit correlation, FX jump upon default.
- Extinguisher on such a swap creates an expected benefit to the dealer due to the default risk of the emerging market sovereign entity.
- Extinguisher: The swap knocks out at zero value if the emerging market sovereign entity or either party defaults.

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# Examples and Questions

- Where does the value come from?
- Can you buy credit protection on an entity from the entity itself?

- The dealer has bought credit protection on the sovereign entity from the sovereign entity itself
- The dealer can sell such protection to a third party

- The sovereign entity has sold default protection on itself
- Reduced default recovery for other creditors

- Why would the emerging market sovereign entity default knowing that it would lose its receivables in the swap upon default?

- Default event needs to reference the underlying bond

- There are potential legal and franchise risks

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# Bigger Picture – High Level Financial/Economic Objectives of a Financial Institution

- Revenue Generation, Income Generation

- Risk Management
  - Market Risk, (Counterparty) Credit Risk/CVA (Credit Valuation Adjustment)
  - Non-Market Risk
  - (Cashflow) Liquidity/Funding (Cost), (Regulatory) Capital

- Enhancing Return on Capital/Equity
  - Capital/RWA (Risk Weighted Asset) Management/Optimization

- More ?

- Non-linear portfolio effects/risks (requiring enterprise-level models)
  - (Counterparty) Credit Risk/CVA, (Cashflow) Liquidity/Funding (Cost), RWA, (Regulatory) Capital, Others

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