Lecture 24

HJM Model for Interest Rates and Credit

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Denis Gorokhov is an Executive Director at Morgan Stanley. During his 9 years at the firm Mr. Gorokhov worked on pricing exotic derivatives with emphasis on credit, counterparty risk, asset-backed securities, inflation, and longevity.

Mr. Gorokhov obtained a number of original analytic results for fixed income pricing problems which are implemented in Morgan Stanley risk management systems. He created a flexible modeling framework used for pricing over 400 different types of exotic derivatives including key transactions in the recent Morgan Stanley history: TARP transaction with US Treasury, purchase of 20% stake in Morgan Stanley by Mitsubishi UFJ Securities, and credit valuation adjustment with monoline insurer MBIA.

Prior to joining Morgan Stanley Mr. Gorokhov worked in the fields of superconductivity and mesoscopic physics and authored 20 papers in leading physics journals. Mr. Gorokhov holds a Ph.D. degree in theoretical physics from ETH-Zurich and spent several years as a post-doctoral researcher at Harvard and Cornell.

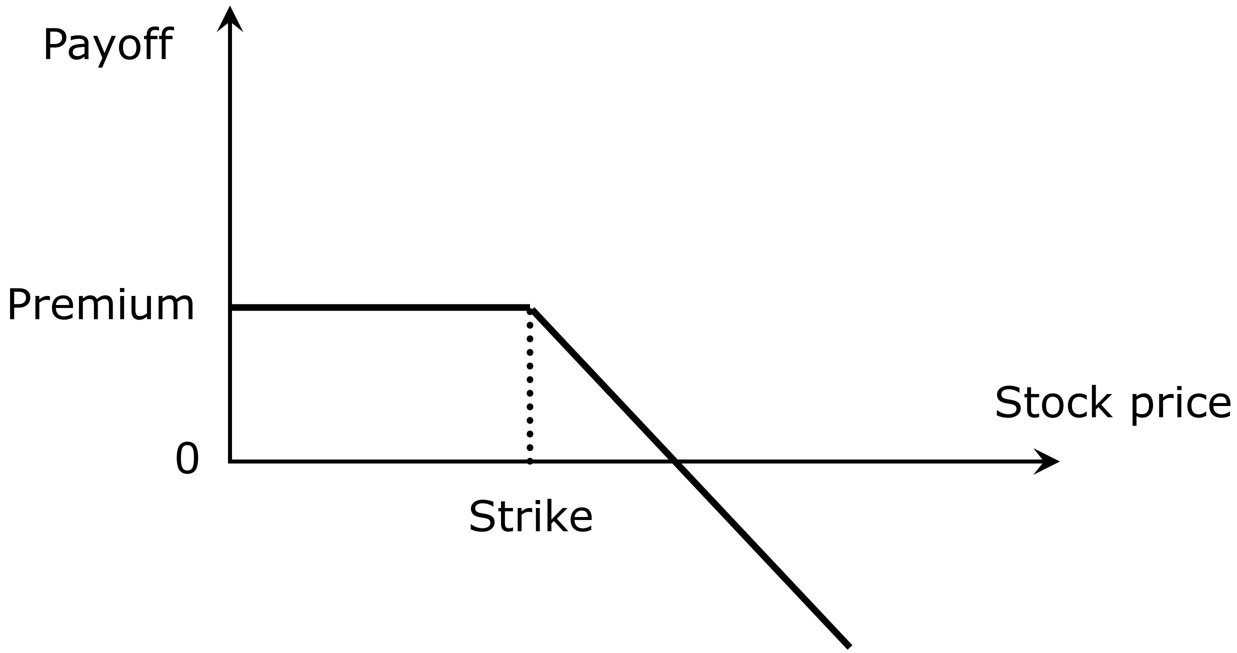
Mr. Gorokhov's comments today are his own, and do not necessarily represent the views of Morgan Stanley or its affiliates, and are not a product of Morgan Stanley Research.

Introduction

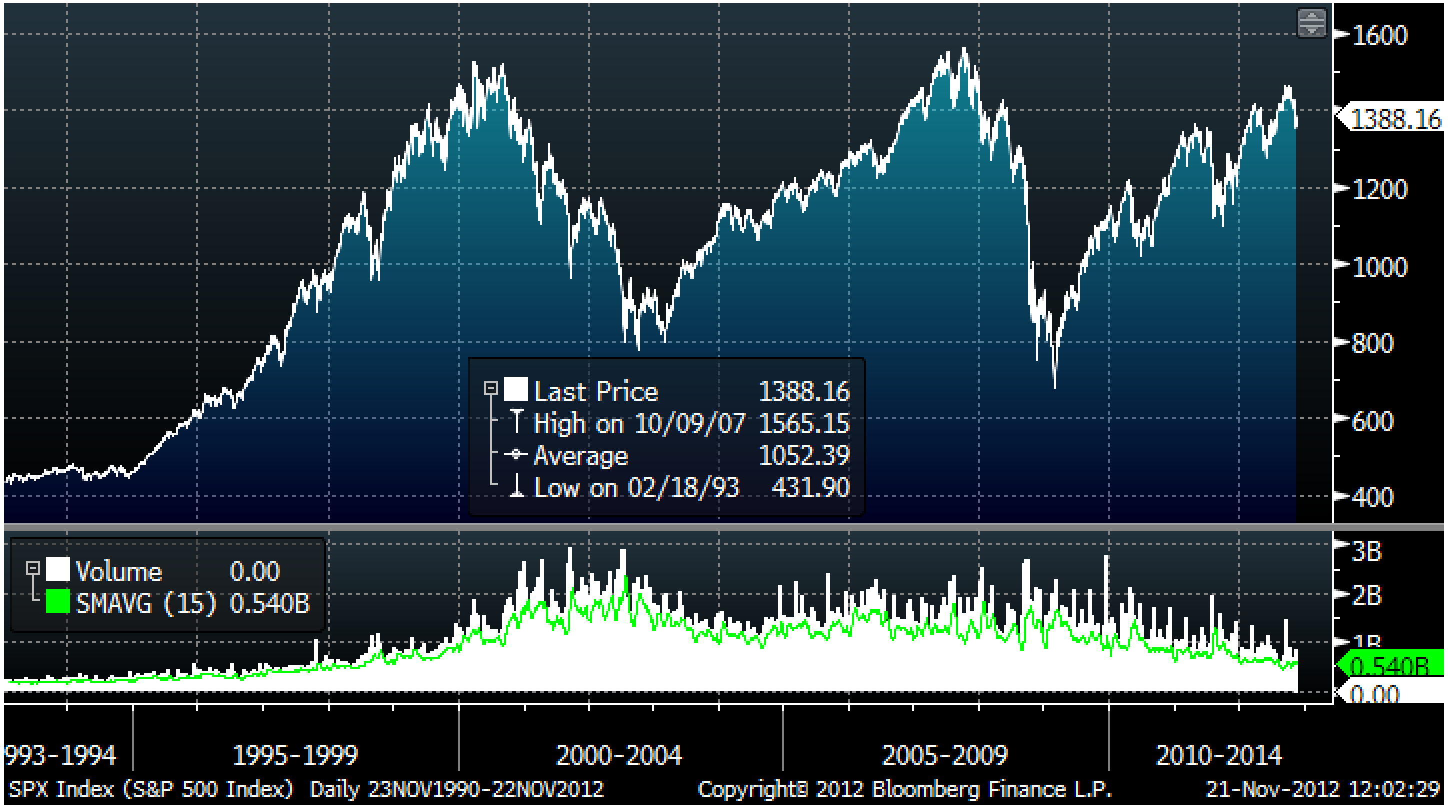
- HJM (Heath-Jarrow-Morton) model is a very general framework used for pricing interest rates and credit derivatives.
- Big banks trade hundreds, sometimes even thousands, of different types of derivatives and need to have a modeling/technological framework which can quickly accommodate new payoffs.
- Compare this problem to that in physics. It is relatively straightforward to solve Schrodinger equation for the hydrogen atom and find energy levels. But what about energy spectra of complicated molecules or crystals? Physicists use advanced computational methods in this case, e.g. LDA (local density approximation).
- Similarly, in the world of financial derivatives there is a very general framework, Monte Carlo simulation, which in principle can be used for pricing any financial contract.
- The HJM model naturally fits into this concept.
- Before discussing the HJM model it is very important to understand how the Monte Carlo method appears in finance. Stock options are the best example.

Dynamic Hedging

- Dealers are trying to match buy and sell orders from clients. It is not always possible and they have to hedge the residual positions.
- In the example below a dealer sold a call option on a stock, the loss may be unlimited. To hedge the exposure the dealer takes a position in an underlying and adjusts it dynamically.



Stock Price Dynamics



S&P-500 index, 1990 -2012

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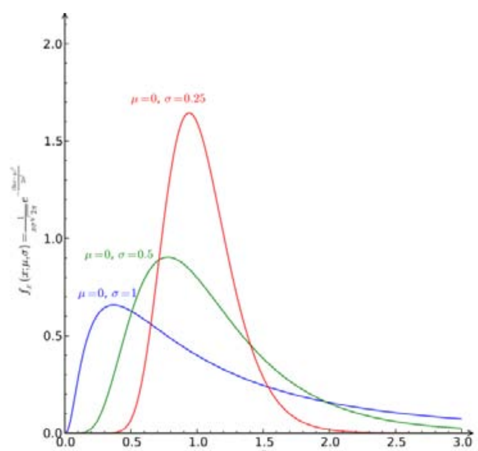
Lognormal Stochastic Process

- Usually stock dynamics is assumed to be a sum of drift and diffusion and a lognormal stochastic process is a good first approximation.
- For this process one can calculate the probability distribution function exactly. This is why the price of the call option in the Black-Scholes model can be calculated analytically.
- The probability distribution function is Gaussian in the log coordinates.

$$dS = \mu S dt + \sigma S dB_t$$

$$P(S_T, S_t; T-t) = \frac{1}{\sqrt{2\pi}\sigma S_t} \exp\left[-(\ln\frac{S_t}{S_T} + (\mu - \frac{1}{2}\sigma^2)(T-t))^2 \frac{1}{2\sigma^2 (T-t)} \right]$$



Black-Scholes Formalism

- Usually derivation of the Black-Scholes equation is based on the result from stochastic calculus known as Ito's lemma.
- Roughly speaking, Ito's lemma says that the derivative of a stochastic function with respect to time has an additional *deterministic* term.
- One can create a portfolio consisting of an option and a position in the underlying (hedge) which is riskless (thanks to Ito's lemma).

$$dC = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} dt, \quad C = C(S,t)$$

$$\Pi = C - \Delta S, \text{ choose } \quad \Delta = \frac{\partial C}{\partial S}$$

$$d\Pi = dC - \Delta dS = r(C - \Delta S)dt, \qquad \text{portfolio is riskless}$$

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0, \qquad r = \text{interest rate}$$

Black-Scholes Miracle

- There are 2 striking facts about the Black-Scholes equation.
- First, the drift of the stock does not show up in the BS equation. It happens because we hedge the option with the stock.
- Second, the risk is eliminated *completely,* i.e. by holding a certain position in underlying we can fully replicate the option. This is closely related to the *deterministic* second-order term in Ito's lemma. It is worth analyzing it in detail.

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

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Ito's Lemma under Microscope

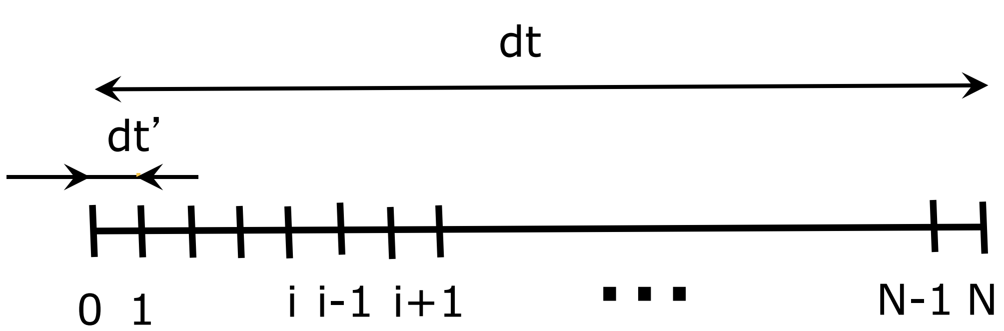
$$S_{i+1} - S_i = \mu S_i dt' + \sigma S_i \varepsilon_i \sqrt{dt'},\ \ \varepsilon_i \sim N(0,1)$$

$$C(S_{i+1}, t_{i+1}) - C(S_i, t_i) \approx \frac{\partial C}{\partial t} dt' + \frac{\partial C}{\partial S}\left(S_{i+1} - S_i\right) + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\left(S_{i+1} - S_i\right)^2,\ \ i = 0 \ldots N-1,$$

$$C(S_{i+1}, t_{i+1}) - C(S_i, t_i) \approx \frac{\partial C}{\partial t} dt' + \frac{\partial C}{\partial S}\left(S_{i+1} - S_i\right) + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 \varepsilon_i^{\ 2} dt'$$

$$C(S_N, t_N) - C(S_0, t_0) \approx \frac{\partial C}{\partial t} dt + \sum_i \frac{\partial C}{\partial S}(S_i, t_i)\left(S_{i+1} - S_i\right) + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 \sum_i \varepsilon_i^{\ 2} dt'$$

- In the limit N->infinity the sum in the last term of the last equation becomes *deterministic* and we obtain Ito's lemma!



Black-Scholes Miracle - 2

- Exercise 1: Prove that $\sum_{i=1}^{N} \varepsilon_i^2,\ \varepsilon_{\mathrm{i}} \sim N(0,1)$ is deterministic in the limit N->infinity.

- Exercise 2: Look up a "proof" of Ito's lemma in Hull's book (J.C. Hull, *Options, Futures, and Other Derivatives*) and find an error (pages 232-233 in 5th edition).

- We see that complete risk elimination in the Black-Scholes model is due to the existence of 2 different time scales dt and dt'. One can derive the BS equation in the limit dt->0, dt'->0, and dt/dt'->infinity.

- From the business point of view this means that we hedge on a small time scale dt' and the profit/loss noise is finite, however as long as the time interval increases to dt the profit/loss noise disappears!

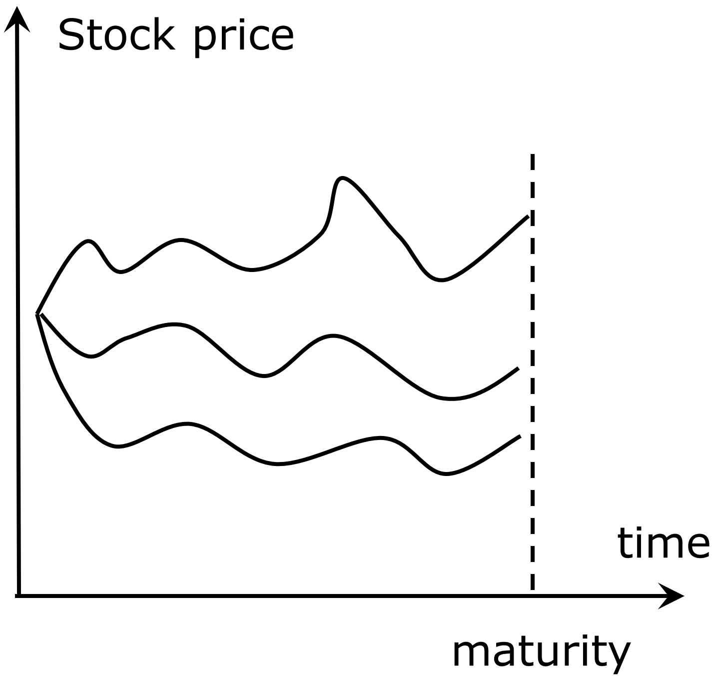
Solving Black-Scholes Equation

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

$$C(S,t) = \exp\left(-r(T-t)\right)\int \frac{dS^{'}}{\sqrt{2\pi}\sigma S^{'}} \exp\left[-(\ln\frac{S}{S^{'}} + (r - \frac{1}{2}\sigma^2)(T-t))^2 \frac{1}{2\sigma^2(T-t)}\right] \mathrm{Payoff}(S^{'})$$

- BS equation is similar to the heat equation and can be solved using standard methods.
- The Green function in the solution above is strikingly similar to the probability distribution function of the lognormal distribution shown above.
- The 2 expressions differ only by the discount factor as well as the stock drift substituted by interest rate *r.*

Interpretation: Monte Carlo Simulation Concept



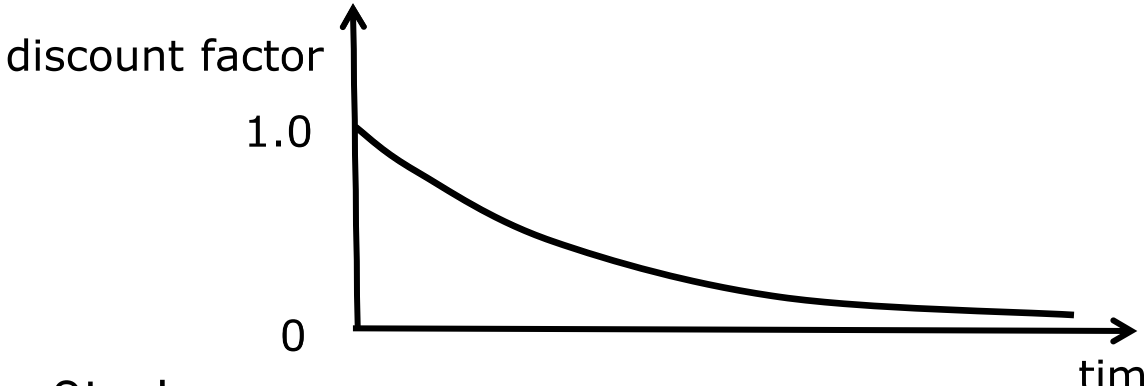
$$dS = rSdt + \sigma SdB_t$$

$$C(S,t) = e^{-r(T-t)} E\left(Payoff\ (S_T)\right)$$

- One needs to simulate different stock paths in the *risk-neutral* world, calculate the average of the payoff, and discount.
- It turns out that this approach works not just for equity derivatives but can be generalized for interest and credit cases as well.
- Every security in the risk-neutral world grows on average with the risk free rate.

Interest Rates Derivatives: Basic Concepts

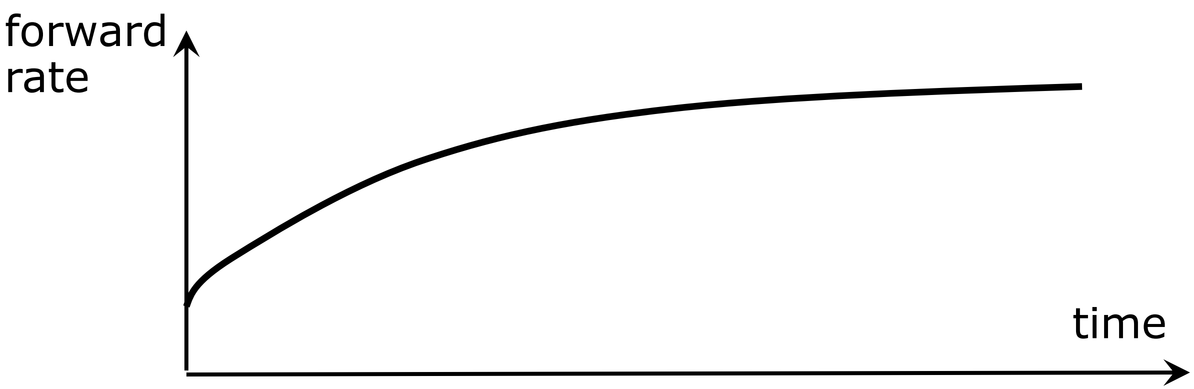
- Businesses borrow money to finance their activity.
- As a compensation lenders charge borrowers certain interest.
- Interest rates fluctuate with time and, similar to the equity case, there exists a market of derivatives linked to the level of interest rates.
- Time value of money: $1 to be paid in 1 year form now is worth less than $1 paid in 2 years form now. For example, if 1- and 2-year interest rates are both equal to 5%, then one needs to invest $1/(1+0.05)= $0.95 today to obtain $1 in 1 year form now and $1/(1+0.05)^2=$0.90 to obtain $1 in 2 years form now.
- It is very convenient to describe time value of money using discount factors.



Forward Rates

- It is very convenient to express discount factors using forward rates.
- Mathematically, the relation between discount factors and forward rates can be expresses as

$$d(t,T) = \exp\left(-\int_t^T f(t,s)\,ds\right)$$

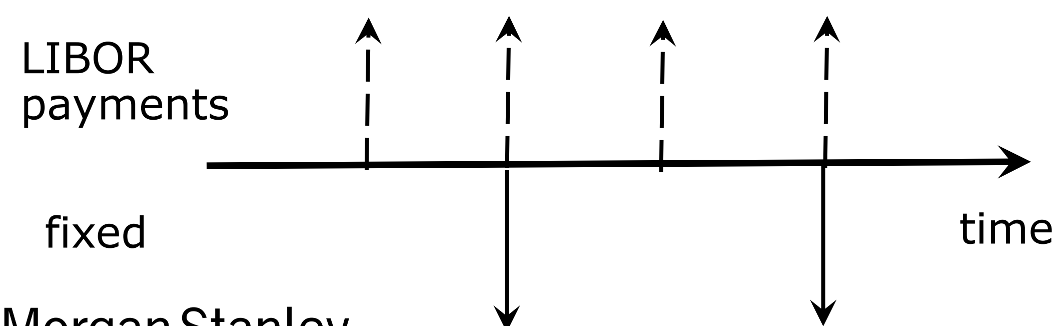


Yield of 10-year US Treasury Note

- Interest rates are extremely low at present.

Libor Rates

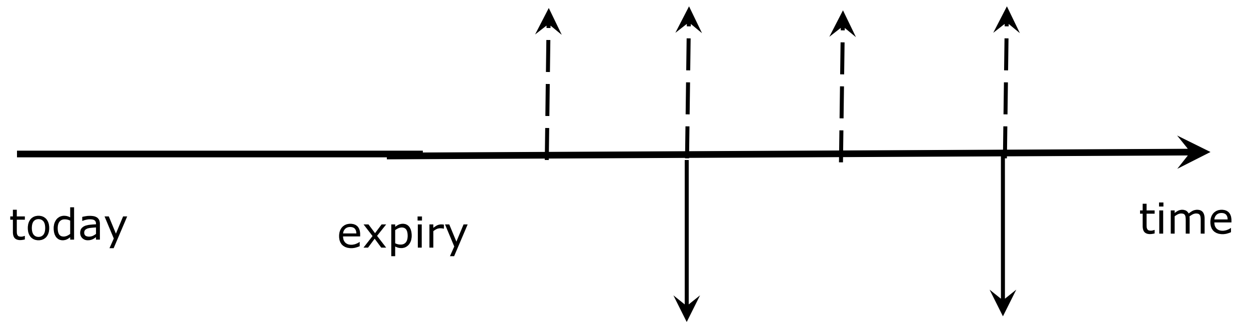
- LIBOR rate is a fundamental rate in the world of interest rate derivatives. This rate is used an important benchmark in the world of lenders/borrowers. Very often borrow rates are quoted as a spread over LIBOR.
- Roughly speaking, USD LIBOR (London Interbank Exchange Offer) rate is the rate at which well rated banks lend US dollars to each other in London for a short term on unsecured basis.
- There exist liquid 1M, 3M, 6M, and 12M LIBOR rates in different currencies.
- *LIBOR swap* is a fundamental interest rates derivative. The first counterparty makes periodic LIBOR payments, while the second one pays predetermined fixed rate.



Standard USD 3M-LIBOR swap

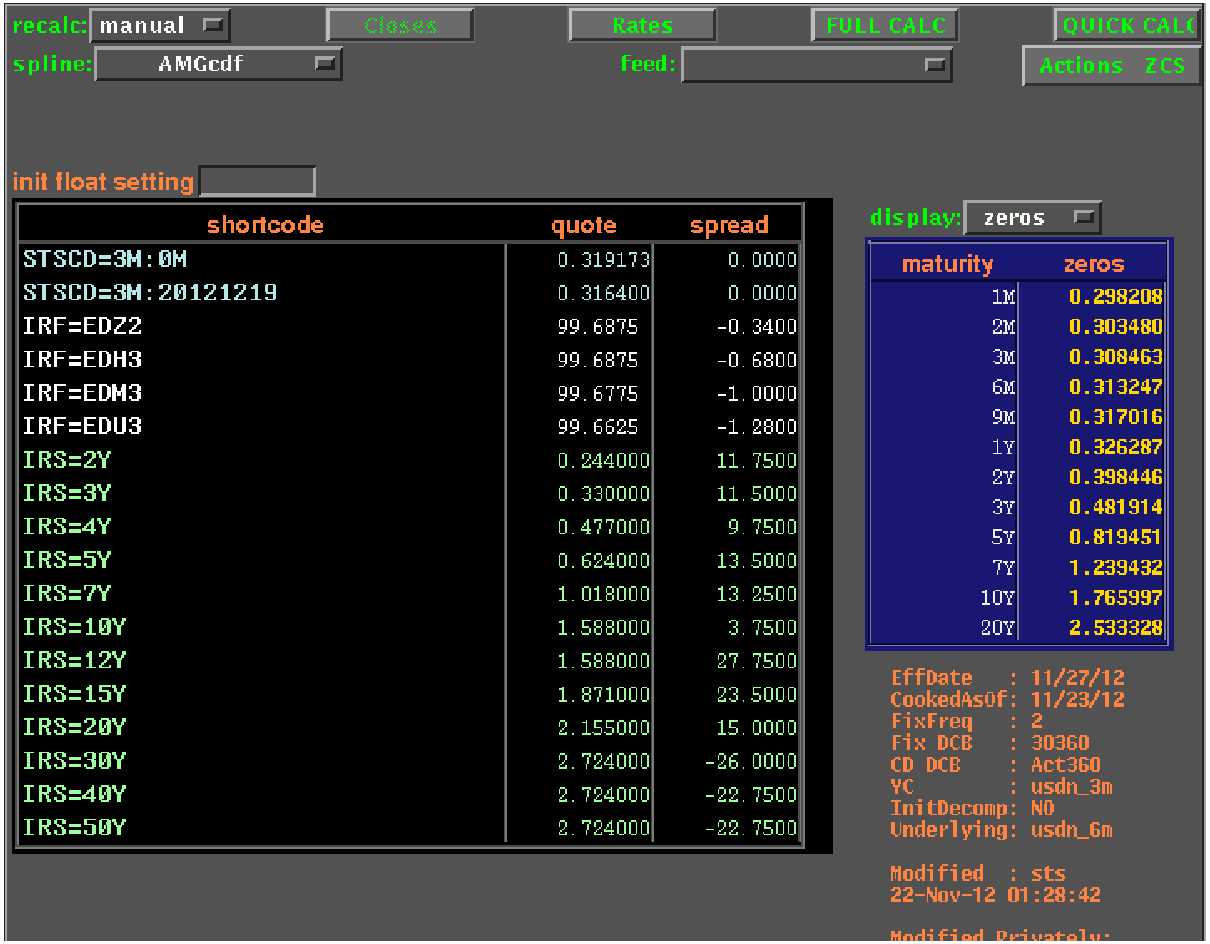
Interest Rate Derivatives

- Besides standard interest rate swaps there exist numerous types of interest rate derivatives. They are used for risk management of interest rate exposure.
- European swaption: An option to enter a forward starting LIBOR Swap.



- Libor caps/floors: Essentially put/call options on LIBOR rate in the future.
- Cancellable swaps: One of the parties has a right to cancel the LIBOR swap.

LIBOR Swap Quotes



shortcode	quote	spread
STSCD=3M:0M	0.319173	0.0000
STSCD=3M:20121219	0.316400	0.0000
IRF=EDZ2	99.6875	-0.3400
IRF=EDH3	99.6875	-0.6800
IRF=EDM3	99.6775	-1.0000
IRF=EDU3	99.6625	-1.2800
IRS=2Y	0.244000	11.7500
IRS=3Y	0.330000	11.5000
IRS=4Y	0.477000	9.7500
IRS=5Y	0.624000	13.5000
IRS=7Y	1.018000	13.2500
IRS=10Y	1.588000	3.7500
IRS=12Y	1.588000	27.7500
IRS=15Y	1.871000	23.5000
IRS=20Y	2.155000	15.0000
IRS=30Y	2.724000	-26.0000
IRS=40Y	2.724000	-22.7500
IRS=50Y	2.724000	-22.7500

maturity	zeros
1M	0.298208
2M	0.303480
3M	0.308463
6M	0.313247
9M	0.317016
1Y	0.326287
2Y	0.398446
3Y	0.481914
5Y	0.819451
7Y	1.239432
10Y	1.765997
20Y	2.533328

Pricing LIBOR Swaps, Discount Curve Cooking

- The LIBOR swap consists of 2 streams: fixed and floating ones. They can be priced if we know the discount factor curve *d=d(t).*
- The present value of a fixed payment *C* paid at time *t* is given by

$$PV_{fixed} = Cd(t)$$

- There is a very neat result: present value of the float leg of a LIBOR swap plus a notional payment at the end is equal to the notional. The PV of the swap receiving fixed, paying float, and maturing at time *T* is

$$PV(T) = \sum_i cNd(t_i) + Nd(T) - N$$

- But where from do we know the discount function *d(t)?* The swap market is very liquid and we know swap rates corresponding to different maturities. One can reverse engineer curve *d(t)* so that the observed swap rates are *fair* rates, i.e. for all swaps *PV(T)=0.*
- This procedure is know as discount curve bootstrapping ("cooking").
- Note: Here, we do not take into account OIS discounting.

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Pricing Interest Rate Derivatives

- In order to price a stock option one needs to know the stock value today as well as future stock dynamics.
- The Monte Carlo simulation framework allows to calculate the price.
- It turns out that the same framework can be used for pricing interest rate derivatives. However, there is a very important distinction: while stock evolution is a point-like process, in the interest rate world we interested in dynamics of a curve, i.e. a one-dimensional object. The problem is more complicated.

	STOCK OPTIONS	IR OPTIONS
INITIAL VALUE	known	not known, curve needs to be cooked
DYNAMICS	point-like object	0ne-dimensional object

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Classes of Interest Rates Models

- Dynamic interest rates models can be divided into 2 large classes: term structure models and short rate models.
- The first class of models (more general) describes the dynamics of the full forward curve, while the second class describes the short term rates only. Here is the list of the most famous short rates models:

$dr_t = \theta_t dt + \sigma dB_t$	Ho-Lee model
$dr_t = (\theta_t - \alpha r_t)dt + \sigma dB_t$	Hull-White model
$dr_t = (\theta_t - \alpha r_t)dt + \sigma \sqrt{r_t}\, dB$	Cox-Ingersoll-Ross (CIR) model

- Note that the short rate and forward rates are related via

$$r_t = f_{tt}$$

HJM (Heath-Jarrow-Morton) Framework

- The HJM model shows how to describe the evolution of forward rates. The starting equation is again a sum of drift and diffusion terms. Note that each forward rate depends on 2 variables *t* and *T*, not just *t* as in the stock options case

$$df_{tT} = \mu_{tT} dt + \sigma_{tT} dB_t$$

- Consider the price evolution of a zero coupon bond (it pays $1 at time T). In the risk-neutral world the price of the bond should grow with the risk-free rate on average

$$Z_{tT} = \exp(-\int_t^T f_{ts} ds) \equiv \exp(-X_t)$$

$$dX_t = -f_{tt} dt - \int_t^T df_{ts} ds = -f_{tt} dt - (\int_t^T \mu_{ts} ds) dt - (\int_t^T \sigma_{ts} dB_t) ds$$

$$\text{Ito's lemma}: \ \mathrm{dZ_{tT}} = de^{-X_t} = -e^{-X_t} dX_t + \frac{1}{2}\left(\int_t^T \sigma_{ts} ds\right)^2 e^{-X_t} dt =$$

$$= f_{tt} Z_{tT} - (\int_t^T \sigma_{ts} ds) Z_{tT} dB_t + \frac{1}{2}\left(\int_t^T \sigma_{ts} ds\right)^2 Z_{tT} dt - (\int_t^T \mu_{ts} ds) Z_{tT} dt$$

HJM Framework - 2

- On the previous slide we have shown that

$$dZ_{tT} = f_{tt} Z_{tT} - (\int_t^T \sigma_{ts} ds) Z_{tT} dB_t + \frac{1}{2}\left(\int_t^T \sigma_{ts} ds\right)^2 Z_{tT} dt - (\int_t^T \mu_{ts} ds) Z_{tT} dt$$

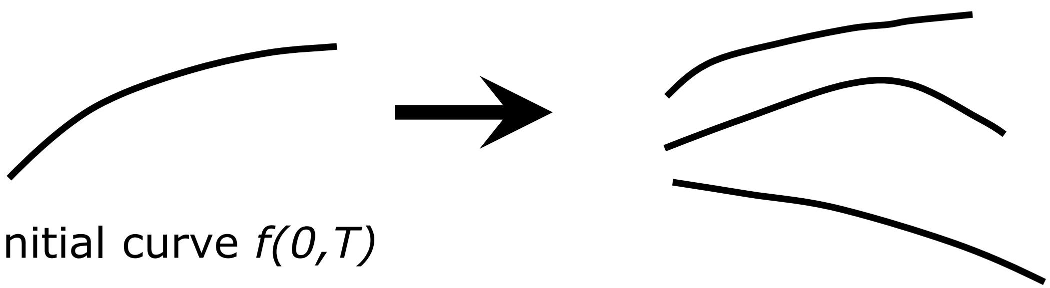
- First of all, the forward rate *f(t,t)* is equal to the short rate *r(t).* Second, in order for the bond price to grow on average with the risk-free rate, the sum of the last 2 terms in the equation above should be equal to zero, i.e.

$$\mu_{tT} = \sigma_{tT} \int_t^T \sigma_{ts} ds$$

- This equation allows to simulate interest rates and price interest rates derivatives in a Monte Carlo simulation.

HJM Model Summary: How to Price IR Derivatives

- Take IR swap quotes and bootstrap the discount curve.
- Obtain forward rates from the discount curve. This is the starting point of the simulation.
- Specify the volatility of forward rates.
- If the volatility is known, the drift can be calculated.
- Start the simulation, stop at maturity, and price the derivative value by averaging and discounting, similar to the stock option case.
- IR volatility can be calibrated from the swaptions market.



initial curve *f(0,T)*

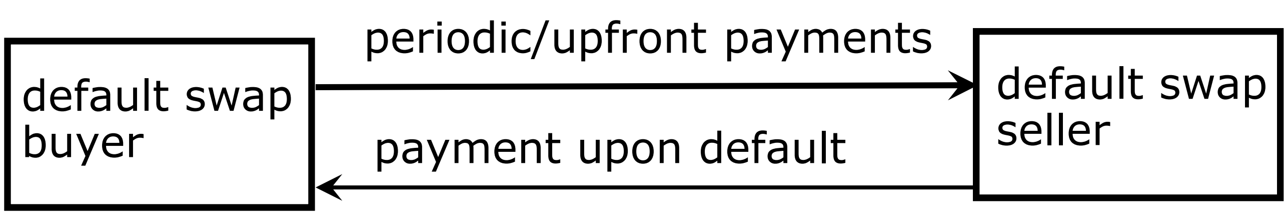
simulated curves at maturity

$$PV_t = E\left(e^{-\int_0^t f_{ss} ds} Payoff(t) \right)$$

$$df_{tT} = \sigma_{tT}\left(\int_t^T \sigma_{ts} ds \right) dt + \sigma_{tT} dB_t$$

Credit Derivatives: a Quick Introduction

- When a borrower issues debt, there is a chance that money will not be returned.
- This type of risk is known as credit risk and needs to be taken into account.
- The higher the default risk for a particular issuer, the higher interest rate is charged by the borrowers.
- The interest rate can vary a lot among different issuers. For example, for 10-year debt, the US government pays 1.7%/year (essentially, default-free rate), Morgan Stanley pays around 5%/year, while the effective yield of bonds issued by Greece is in the range 25-30%.
- There exists a large credit derivatives market. The most important instrument is credit default swap.



- In the event of default of a reference entity, seller pays the loss on a bond.

Survival Probabilities and Hazard Rates

- It is very convenient to describe credit derivatives in terms of implied survival probabilities. Let us illustrate this using a credit default swap example.
- Assume that credit default swap protects against default on a bond with notional *N* and recovery *R*. Assume for simplicity that interest rate is zero.

$$cN = (1-R)(1-p)N$$

$$c = \text{upfront coupon}, N = \text{notional}, R = \text{recovery}, p = \text{survival prob.}$$

$$p = 1 - \frac{c}{1-R}$$

- It is very convenient to parameterize survival probabilities with hazard rates. Similar to the IR case on can write

$$p(t,T) = \exp(-\int_t^T h(t,s)ds)$$

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Pricing Credit Derivatives

- Remember that in the IR case one could price all derivatives using discount curve *d(t)* whose meaning is the value of $1 paid at time *t* form now.
- In the world of credit derivatives there is a notion of risky discount factors, i.e. the value of $1 paid in the future subject to a possible default of a reference entity.
- Similar to the IR case one can "cook" survival probabilities from CDS market.
- The present value of a CDS paying a loss on a bond can be written as follows

$$PV = \sum_i aNd(t_i)p(t_i) - (1-R)N\sum_i (p_{i-1} - p_i)d_i, \text{ a} = \text{running coupon}$$

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Dynamic Hazard Rate Model

- In the case of credit default swaps pricing is relatively simple.
- There are credit-linked instruments depending not only on survival probabilities but also on their volatility, e.g. callable bonds.
- Correct modeling of interest rate and credit dynamics allows to make optimal call decisions.
- Here is an example:

corporation X issues a 10y \$100MM bond

risk free rate = 2.00%

credit spread = 3.00%

effective spread = 5.00%

in 3y the market changes

risk free rate = 1.50%

credit spread = 1.50%

effective spread = 3.0%

the company decided to refinance,

savings are 2% * \$100MM * 7years = \$14MM

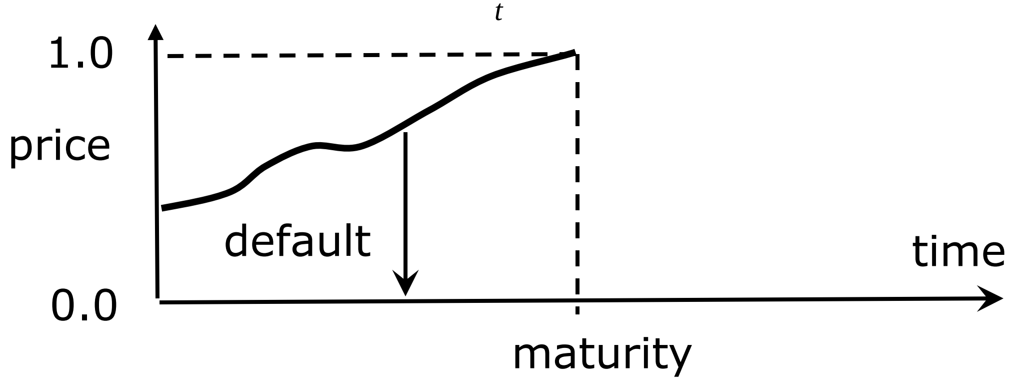
Modeling Dynamics of Hazard Rates

- Similar to the IR case we assume that dynamics of forward hazard rates can be described by a sun of drift and diffusion terms.
- In order to describe dynamics of hazard rates it is very convenient to consider price evolution of a zero coupon bond with no recovery.

$$df_{tT} = \mu_{tT} dt + \sigma_{tT} dB_t$$

$$dh_{tT} = \mu^h_{tT} dt + \sigma^h_{tT} dB^h_t$$

$$Z_{tT} = 1_{\{t<\tau\}} \exp(-\int_t^T \left(f_{ts} + h_{ts}\right) ds), \tau = \text{default time}$$



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Drift Condition for Hazard Rates

- Exercise 3: Derive a drfit expression for the credit HJM model.
- Hint: Consider dynamics of a zero coupon bond and average over the default event. The resulting drift in this case should be equal to the risk-free rate. This leads to a condition for the drift of forward hazard rates.
- If there are no correlations between IR and hazard rates, the drift is given by the equation similar to the IR case.

$$dh_{tT} = \sigma_{tT}^{h} \int_{t}^{T} \sigma_{tT}^{h} ds + \sigma_{tT}^{h} dB_t$$

- In summary, we have shown how to price exotic derivatives using Monte Carlo simulations. One needs to write stochastic differential equations with the correct drift (to make sure all assets grow with risk-free rate on average), simulate equity/IR/credit paths, and (in the European option case) average/discount payoff

Example: Structured Notes

- Investors are looking for attractive yields on their capital.
- Nowadays investing into Treasury bonds is not very attractive. The 10 year bond generates just 1.7% per year. After taxes the investor is left with just 1.1% return which is well below current inflation rate (1.5-2%).
- Investors can increase their return by buying corporate bonds and, hence, bearing credit risk. In this case one can easily generate a 5-6% nominal return before taxes (Morgan Stanley case). But can we go even higher than that without bearing too much credit risk?
- Structured note is a bond issued by institutions whose coupon can be very exotic. Here is a typical example.

$$\text{corporation X issues a 10 year \$100MM structured note}$$

$$\text{coupon} = 10\% * \text{number of days conditon is satisifed/total number of days}$$

$$\text{condition} = (\text{30-year swap rate} > \text{2-year swap rate}) \,\&\&\, (\text{S\&P-500 index} > 880)$$

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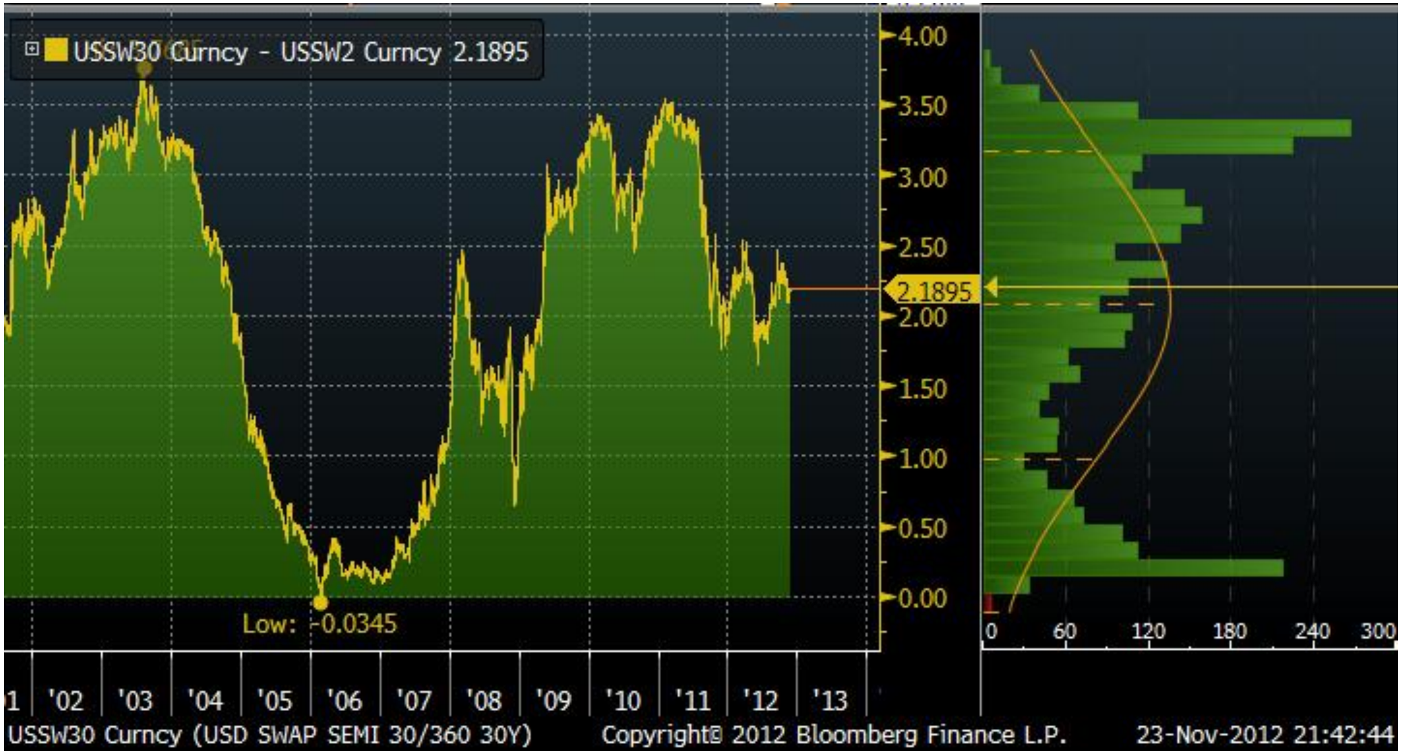
Structured Notes: Coupon Enhancement

- The bond holder bears tail risk: The historical probability of events is low but the market implied probability of them is significant, so that the investor can obtain a coupon enhancement.
- What will the institution who sold the bond do? First, the note is priced using a Monte Carlo simulation which was described in the presentation. Next, deltas with respect to equity, rates, and credit will be calculated and the note will be hedged accordingly.
- For the described structured note the breakdown of the condition implies a very severe recession.

30y swap rate – 2y swap rate



- During the last decade the 30y-2y difference was negative only for a few days on Feb 2006 but the market implies that for the next decade it will be the case for nearly 80% of days.
- Similarly, S&P-500 was above 880 for 94% of days, however the implied frequency is only about 75% for the next decay.

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