Lecture 23

Quanto Credit Hedging

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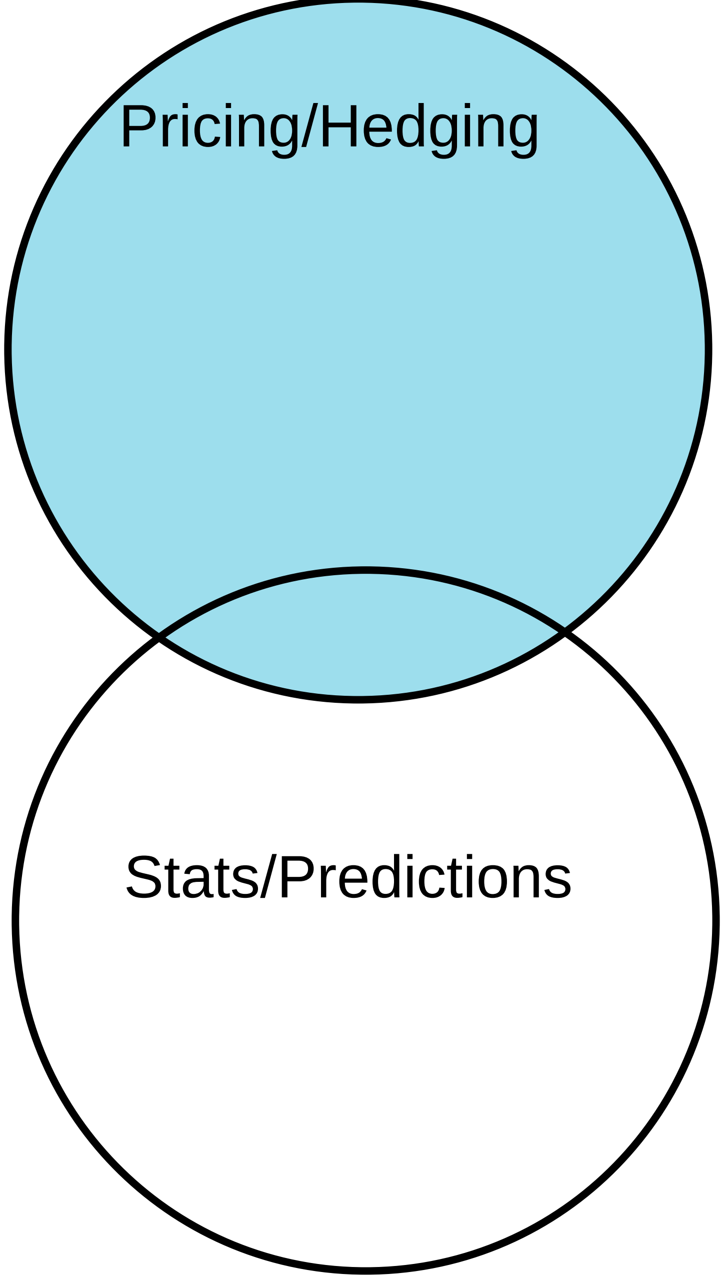
Mr. Andreev's comments today are his own, and do not necessarily represent the views of Morgan Stanley or its affiliates, and are not a product of Morgan Stanley Research.



Big Picture

- Topic: Pricing/Hedging
- Basic concepts
  - Main focus: FX
    - Interest Rates
    - Credit
- Mathematical Techniques
  - Risk-neutral pricing through expectations
  - Jump processes
- Financial applications
  - Sovereign defaults and currencies (PIIGS and EUR)
  - Quanto Credit

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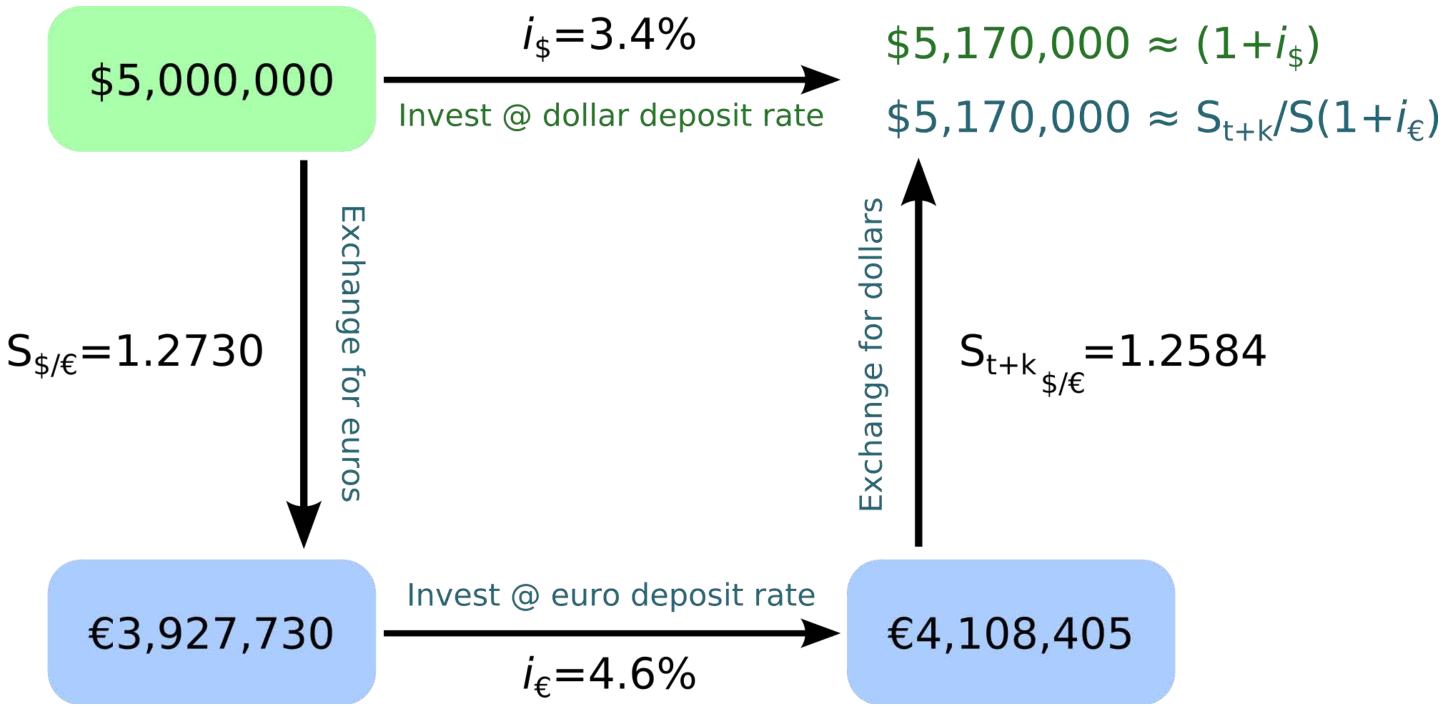
FX – Foreign Exchange

- Spot FX (USD/EUR)– Current exchange rate in USD for 1 EUR
  - Alternatively, the price in USD of 1 EUR
- Denote spot FX rate by S. S is often modeled as a stochastic process, a Brownian motion with drift

FX forwards and interest rate parity

- Forward FX contract is an agreement to exchange X EUR for Y USD at time T in the future. Y/X is the Forward Rate $F_{tT} = S_t e^{\left(r^f - r^d\right)(T-t)}$
- The fair forward FX rate is determined by interest rates and spot FX
- Continuous interest rates:



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Interest Rates and compound interest

- Risk free instantaneous interest rate r
  - Useful modeling approximation, does not quite exist in reality
  - Investing money B at time 0 earns a risk free return so that at time T in the future you have B*exp(rT)
  - Each currency has its own interest rates

$$\frac{dS_t}{S_t} = \left(r^f - r^d\right)dt + \sigma dW_t$$

- Same as Black Scholes, solving the SDE we get

$$S_t = S_0 e^{\left(r^f - r^d - \sigma^2/2\right)dt + \sigma W_t}$$

$$E[S_t] = S_0 e^{\left(r^f - r^d\right)dt}$$

- In this model, we can price various FX derivatives, such as FX forwards or options

FX Betting Game

- Assumptions
  - USD/EUR Spot is 1
  - USD/EUR FX forward in 1 month is 1
- Bet B
  - If USD/EUR is more than 1 in 1 month, you **lose**
  - If USD/EUR is less than 1 in 1 month, you **win**

Pop Quiz

Which game is better?

A. Payoff A
B. Payoff B
C. Both are equal

Outcome	Payoff A 	Payoff B 
USD/EUR > 1	-100 USD	-100 EUR
USD/EUR < 1	+100 USD	+100 EUR

FX Betting Game – Scenario Analysis

- Run some scenarios and compare the payoff of each bet

Scenario (in 1M)	Bet A	Bet B	Bet A – Bet B
USD/EUR = 1.25	-100 USD	-100 EUR	+25 USD = +19 EUR
USD/EUR = 0.75	+100 USD	+100 EUR	+25 USD = +31 EUR

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- Run some scenarios and compare the payoff of each bet
  - **Bet A is better each time**, even though both payoffs are symmetric!

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- Lesson – the currency of the payoff matters when winning the bet is correlated with FX – the game is not symmetric anymore.

Reality Check: Italy Bonds

- Italy issues bonds in both EUR and USD (among others), total $1.3 trillion of bonds!
  - Cross-default: bonds of all currencies default together
- Potential reasons to issue in USD
  - Access to other, potentially bigger pool of investors
  - Italy is an exporter, much revenue is in USD
- Credit spread is the premium required for borrowing over the benchmark
  - Higher rates to borrow EUR than Germany
  - Higher rates to borrow USD than USA
- Questions
  - Which currency does Italy prefer to raise money?
  - Which currency investors prefer?

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Pop Quiz

Italy pays higher credit spread premium in

A. USD
B. EUR
C. Equal in both

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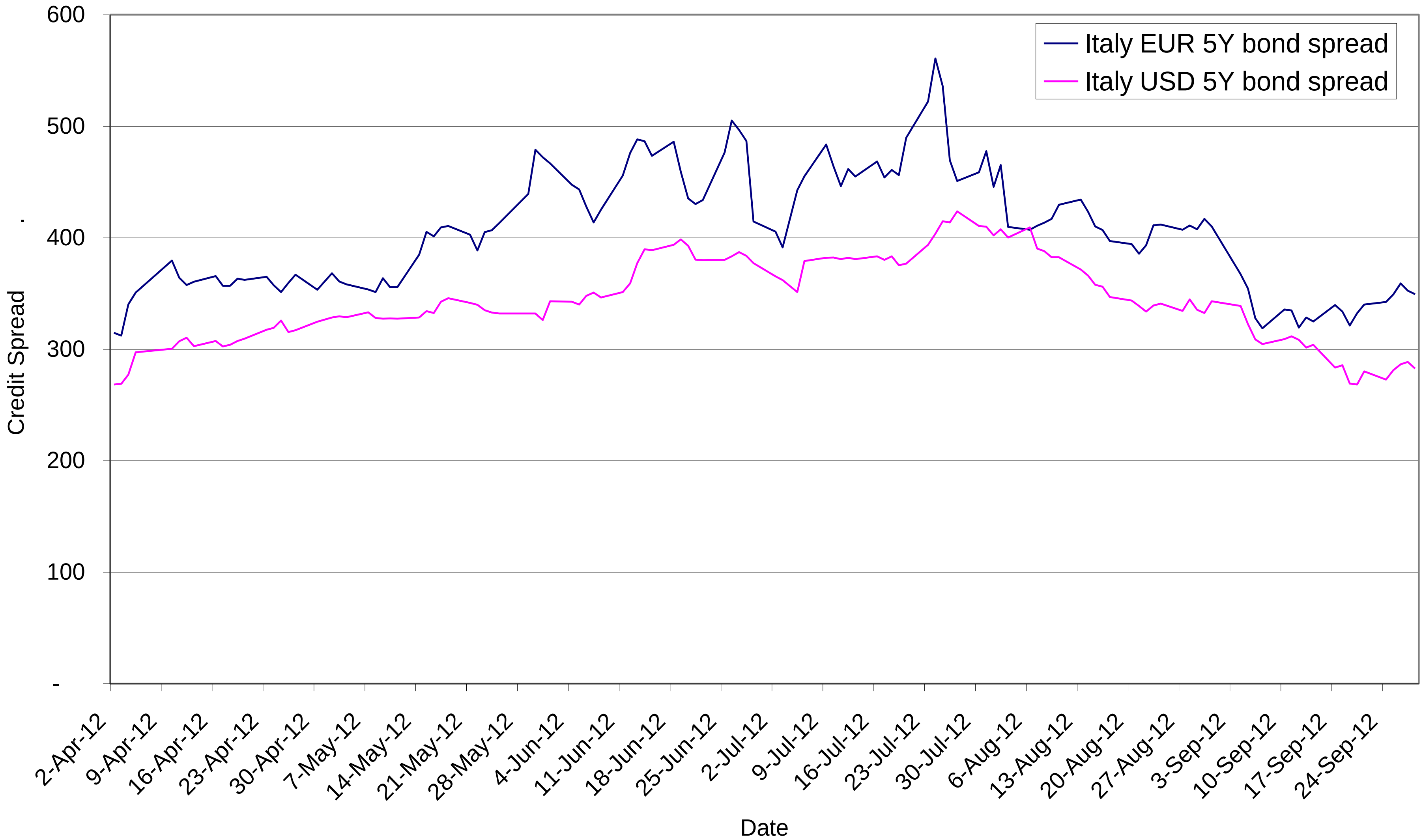
Reality Check: Italy Bonds

- Italy credit spread and EUR/USD FX rates are **volatile**
- Pricing questions
  - How do you compare the value of EUR bonds vs. USD bonds?
  - How do you come up with a strategy to replicate USD bonds with EUR bonds?
- Similarities to the FX betting game
  - Is value of the payoff currency correlated with the payoff event?
- Theoretical pricing argument
  1. Analyze the payoff of both instruments
  2. Use math finance to **price** (replicating trading strategy) bonds
  3. Obtain intuitive understanding (rules of thumb) from the results

Italy Credit Spreads: EUR vs. USD (adj. for basis)



Replication/Arbitrage strategy

- Bonds regimes: **performing** and **non-performing (default)**
- Complete market = replicating strategy exists
- Goal is to replicate one bond with another in both regimes
- Example:
  - Two zero-coupon bonds (**E** and **U**), same maturity, pay 100 at maturity T.
  - **U** pays 100 USD, **E** pays 100 EUR
  - Price of U=Pu; Price of E=Pe;
  - Spot FX=St; FX forward to T=Ft (USD/EUR)
- Trivial potential arbitrage strategy of 1000 E bond with U bonds (initial price= 1000*Pe)
  - Sell 1000*Ft U bonds with proceeds 1000*Ft*Pu
  - Buy 1000 E bonds at cost 1000*Pe
  - Enter into long USD FX forward (will be buying USD and selling EUR) for 100,000 EUR for maturity T at 0 cost

Potential Strategy: Payoff

- Strategy Payoff:

You

Market

100,000 EUR ← *EUR Bond*

100,000 EUR → (*FX Forward*)

100,000*Ft USD ← (*FX Forward*)

100,000*Ft USD → *USD Bond*

  - Net payoff: 0!

- Is the strategy an arbitrage if initial cost is not 0, i.e. if $Ft*Pu \neq S*Pe$?

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Replication/Arbitrage strategy cont'd

- Arbitrage: Start with 0 money, make money with non-zero probability
- What happens if bond defaults?
  - Each bond pays the same % of notional, called recovery rate, typically much less than 100%
  - For sovereign issuers, expected recovery rate around 25%
- Strategy payoff in case of default with 25% recovery rate:
  - Receive 25,000 EUR from bonds E
  - Convert 25,000 EUR to 25,000*Ft USD using the FX forward (*)
  - Pay 25,000*Ft USD on short bond U position
  - Still left with 75,000 EUR FX forward!
  - If EUR weakens strongly on default, you win big! If it strengthens, you lose big!
- Strategy is NOT an arbitrage, not effective in replicating cashflows in all scenarios

Argentinean Peso/USD Devaluation on Credit Default

Applying mathematical finance

- Can we do better? Need a model that captures essential features of the market
- Mathematical model -> hedging/replication strategy
- Essential Model Features
  - Possibility of a credit event (default)
  - FX changes on default
- Complete market
  - Number of hedging instruments ≥ number of model stochastic variables / sources
  - Pricing gives a replicating strategy, means that the price is unique
- In practice, we try to model complete markets, even if some of the instruments do not actually trade

How do we use a model in trading?

1. Define the model with the desired dynamics
   - Check that the market in the model is complete
   - Check that it represents the salient features of the market
2. Price all the instruments
   - Generally, you need to solve a stochastic equation
     - Analytically
     - Numerically (PDE solvers, Monte Carlo simulations)
3. The sensitivity of the target instrument price to the prices of the replicating instruments gives you the hedging ratios (how much to buy or sell for the replicating portfolio)

$$\text{Hedge Ratio of instrument A} = \frac{\partial\,\text{Price(target)}}{\partial\,\text{Price(hedge A)}}$$

A Basic Credit Model

- Goal: Model the default event of an issuer of bonds
- Let's label the time of default τ, a random time
- Model τ as the arrival time of the first jump in a Poisson process

$$P_t(T) = \Pr(\tau > T \mid \tau > t) = e^{-h(T-t)}$$

- Make the assumption that the intensity h (hazard rate) is constant and τ>t
- Probability density that default happens at time T>t

$$\phi(t,T) = \lim_{\Delta T \to 0} \frac{P_t(T) - P_t(T+\Delta T)}{\Delta T} = -\frac{\partial P_t(T)}{\partial T} = he^{-h(T-t)}$$

- Corollary: Current probability density of default between time t and t+dt

$$\phi(t,t) = h$$

Minimal FX jump-on-default model: Definition

- FX jumps by a fixed multiplier on default

$$S_{\tau^+} = S_{\tau^-} e^J, J \in (-\infty, \infty)$$
$$\ln(S_{\tau^+}) = \ln(S_{\tau^-}) + J$$

- Define jump-on-default Poisson process with intensity h

$$N_t = 1_{t>\tau}$$
$$P(\tau > t) = e^{-ht}$$

- Define FX dynamics

drift jump

$$d\log(S_t) = \mu_t dt + J dN_t$$

Show that we need $\mu_t = h(1-e^J)1_{t\leq\tau}$, so that $F_{tT} = E[S_T] = S_t$

- Proof in accompanying notes and on the board

Overview

- So far…
  - Defined a dynamics for log(S) with jump on default
  - Derived the probability density of the FX rate in the future
- Next…
  - Derive the dynamics of S
  - Price EUR and USD bonds
  - Derive hedge ratios
  - Construct a replicating portfolio
  - Check that it is indeed replicating
  - Does the result give any intuition, rules of thumb for trading?

Minimal FX jump-on-default model: Derivation

- Log(S) dynamics

$$d\ln(S_t) = h(1-e^J)1_{t<\tau}\,dt + J\,dN_t$$

- Applying Ito's lemma, we get the dynamics of the FX rate S

$$\frac{dS_t}{S_t} = h(1-e^J)1_{t<\tau}\,dt + (e^J - 1)dN_t$$

- Solve the SDE

$$S_T = S_t e^{\left[h\tau(1-e^J)+J\right]1_{t\geq\tau}} e^{hT(1-e^J)1_{t<\tau}}$$

Details of the derivation in the notes and on the board

Back to Bonds: Pricing

- Two zero coupon, zero recovery bonds. One pays 1 USD, the other 1 EUR.
- Using the model to hedge
  1. Price both EUR and USD bonds in USD currency with the model
  2. Ratio of prices gives the ratio of notionals in the hedge portfolio
- USD bond price is ${P_t}^{USD} = e^{-hT}$
- EUR bond price is ${P_t}^{USD} = e^{-hTe^J}$
- Construct portfolio at t=0
  – Sell 1 USD bond
  – Buy $\frac{e^{-hT(1-e^J)}}{S_0}$ EUR bonds
  – Portfolio value at t=0 is 0

Profit and Loss after ΔT

- Some time ΔT later we have

$$P_0^{USD} = e^{-h(T-\Delta T)} \;;\; P_0^{EUR} = e^{-he^J(T-\Delta T)} \;;\; S_{\Delta T} = S_0 e^{h\Delta T(1-e^J)}$$

- The value of the two positions is

Position	USD Bond	EUR Bond
Value	$-e^{-h(T-\Delta T)}$	Number of bonds: $\frac{e^{-hT(1-e^J)}}{S_0}$; Price of each bond: $e^{-he^J(T-\Delta T)}$; FX rate S_T: $S_0 e^{h\Delta T(1-e^J)}$ $\frac{e^{-hT(1-e^J)}}{S_0} e^{-he^J(T-\Delta T)} S_0 e^{h\Delta T(1-e^J)} = e^{-h(T-\Delta T)}$

- Final portfolio is hedged both in default and non-default
- Hedging strategy
  - Dynamic (i.e. hedge is rebalanced continuously)
  - Depends on credit riskiness or default probability
  - Depends on the size of the jump on default

What if R>0?

- If recovery R>0 , prices are

$$P_t^{USD} = R + e^{-hT}(1 - R)$$

$$P_t^{EUR} = R + e^{-hTe^J}(1 - R)$$

  - Exact replication is not possible

- See class notes for model and hedging strategy details

- Even in such simple model and simple instruments, hedging is not trivial!

Final Paper

- One step further
  - Add diffusive dynamics to the FX process to make it jump-diffusion model

$$\frac{dS_t}{S_t} = h\left(1 - e^J\right)1_{t<\tau}\, dt + \left(e^J - 1\right)dN_t + \sigma dW_t$$

- Possible final paper topic
  - Price zero coupon/zero coupon bonds in USD and EUR in this jump-diffusion model
  - Determine the dynamic hedging strategy
    - Two sources of risk, so need at least 2 hedging instruments. FX forwards are a great candidate.
  - Bonus: Check results with Monte Carlo simulation
  - Follow the presentation in the Reference slide, plus references therein

Real Life

- To apply in finance, need to introduce more sources of risk
  - Stochastic interest rates (term structure model)
  - Stochastic hazard rates (default probabilities), see next lecture
  - Stochastic FX
  - Correlation among the diffusive portions of interest rates, hazard rates, and FX
  - Uncertain and non-zero recovery values
  - Simultaneous modeling of multiple credits
- Analytic solutions are not possible, always use Monte Carlo
  - Default events are rare events, so not desirable for simulation
  - Need to be able to integrate the payoff over the default event distribution only, Monte-Carlo simulate hazard rates, interest rates, and FX

Other Applications

- Modeling stocks with jumps. Very often can give a better explanation of volatility skew at short expiries.
- Quanto credit default swap – same as insurance, but the insurance notional is in a different currency from the underlying bond.
- Same technique can be applied to jumps in interest rates on default

References

- Jump Diffusion Models: More rigorous derivation of hedging in the context of stock prices. Could be a good source for your paper. http://www.john-crosby.co.uk/pdfs/JCrosby_OxfordJune2012_Levy.pdf
- Credit Models: A rigorous discussion of credit and explanation of the hazard model. See attached paper by Ruthkowski, as well as next lecture.

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