Lecture 10

Regularized Pricing and Risk Models

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Plan for today

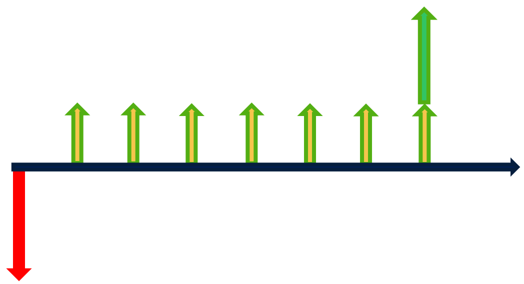
- Bonds
- Swaps
- Yield curve
- Regularized yield curve models
- Regularized volatility surface

Bonds

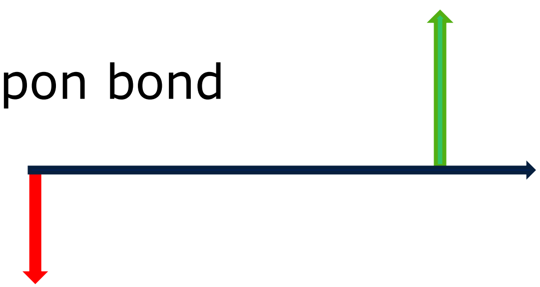
- A debt security
- Borrower issues bonds to obtain funds
- Investor purchases bond to earn return
- Typical bonds include fixed periodic coupon payments plus face value at maturity
- Zero coupon bonds – only face value at maturity, no coupons
- There are perpetual bonds – infinite regular coupon payments, but no face value, as the bonds never mature

Bond Cashflows

- Fixed rate bonds (periodic coupon payments and principal at maturity)



- Zero coupon bond



- Sum of future cashflows is not equal to bond price because future cashflowas are less valuable (time value of money)
- Discount factor

Bond Price

- Present price of the bond should be the sum of present values (PV) of future cashflows

$$P = \sum_{i=1}^{N} cF\Lambda_i + F\Lambda_N$$

Where ***P*** – fair bond price
F – face value of bond
$\mathbf{\Lambda_i}$ - discount factor for payment date *i*
c – coupon rate
N – number of coupon periods

- Need model for discounting $\mathbf{\Lambda_i}$

Yield to Maturity

- Use one parameter y – yield to maturity to compute all discount factors

$$\Lambda_i = e^{-yt_i}$$

$$P = e^{-yt_1}cF + e^{-yt_2}cF + \ldots + e^{-yt_N}cF + e^{-yt_N}F$$

$$P = \sum_{i=1}^{N} e^{-yt_i} C_i$$

Where *y – yield to maturity*
t – future time of payment, years
$\mathbf{C_i}$ - *i-th cashflow*

- Continuous compounding case
- Assumed constant y for all ti

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Bond Duration

- Sensitivity of bond price (ln(P)) to bond yield

$$d = \frac{1}{P}\frac{\partial P}{\partial y}$$

$$d = -\frac{1}{P}\sum_{1}^{N} t_i e^{-yt_i} C_i = -\frac{\sum_{1}^{N} t_i e^{-yt_i} C_i}{\sum_{1}^{N} e^{-yt_i} C_i}$$

Where *d – bond duration*
C_i - *i-th cashflow*

- Duration = "weighted time"
- Duration of zero coupon bond always equals to its maturity
- Duration of regular coupon bond is always less then its maturity
- As there is just one y for all payment dates, the duration is a sensitivity to "parallel" move

Bond Convexity

- Second derivative of bond price to bond yield

$$c = \frac{\partial^2 P}{\partial y^2}$$

$$d = \sum_{1}^{N} t_i^2 e^{-y t_i} C_i$$

Where *c – bond convexity*
Ci - i-th cashflow

- Duration is good measure for price changes for small variation in yield
- Second derivative needed for large changes in yields
- Convexity is always positive

Fixed-vs-float swap analytics

- Valuing fixed and float legs of the swap

$$PV\_fixed = \sum_i C\delta_i \Lambda_i = C\sum_i w_i$$

$$PV\_float = \sum_i r_i \delta_i \Lambda_i = \sum_i r_i w_i$$

$$PV\_fixed = PV\_float$$

$$C = \sum_i r_i w_i / \sum_i w_i$$

Where ***C*** – Swap rate (fixed leg coupon)
$\mathbf{\Lambda_i}$ - discount factor for payment date *i*
δ_i – day count fraction
r_i – forward rate (floating rate of future payment)

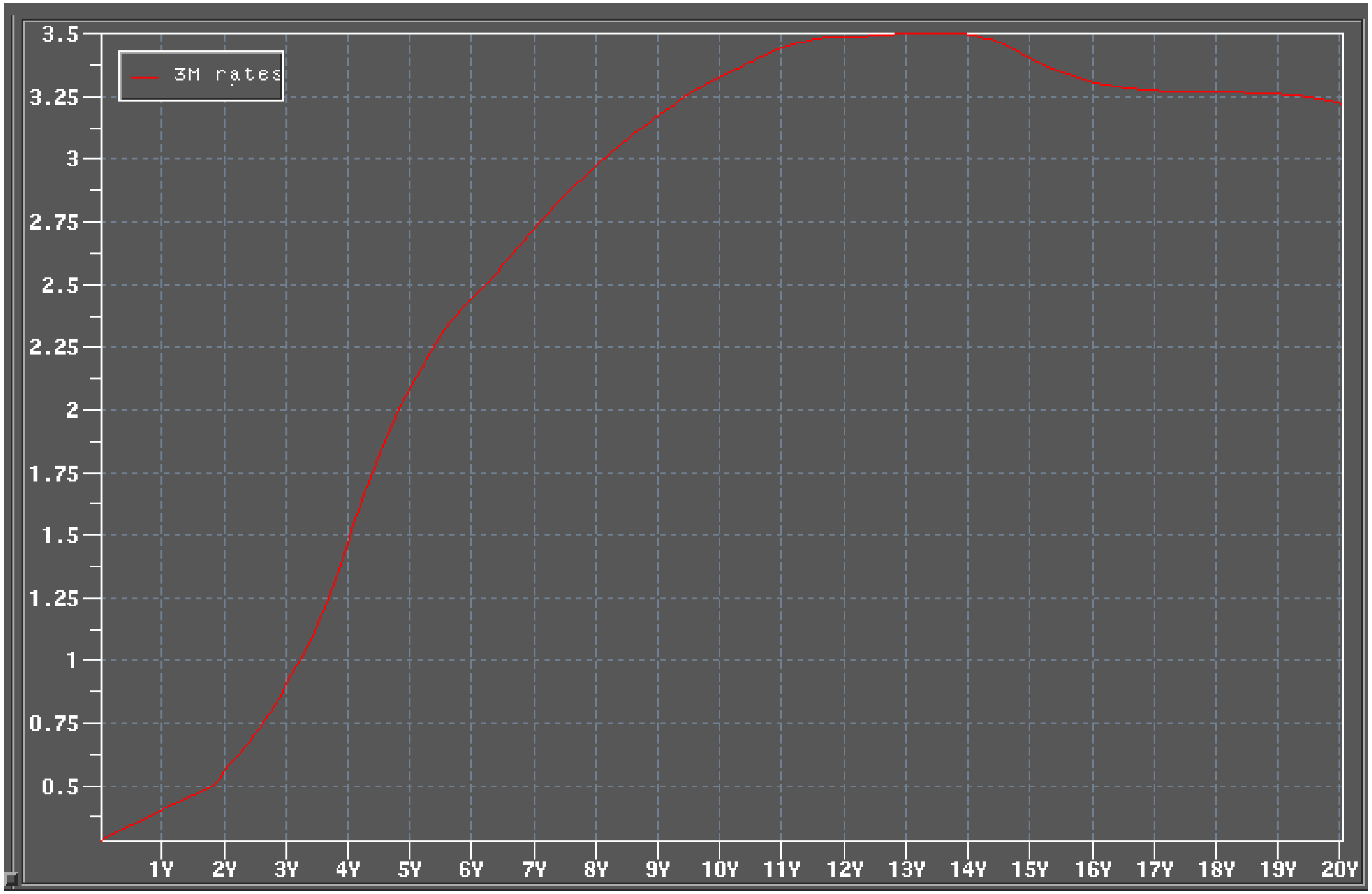
- Swap rate is weighted sum of forward rates (assumed same frequency of payments of fixed and floating legs)
- Swap can be hedged with bond

Constructing Yield Curve

- Select input instruments
- Choose interpolation
  - Interpolation space (daily forward rates, zero rates, etc.)
  - Spline (piecewise-constant, linear, tension spline, etc.)
  - Knot points and model parameters
- Calibrate = solve for spline parameters such that input instruments are re-priced at par

Yield Curve Graph

- Graph of 3M forward rates



Bond Spread to Yield Curve

- We have curve now. So we can use can compute more accurate discount factors Λ_i , rather than relying on "flat" curve with same **y** for all cashflow dates

- Need extra parameter bond spread **s** to match with bond price

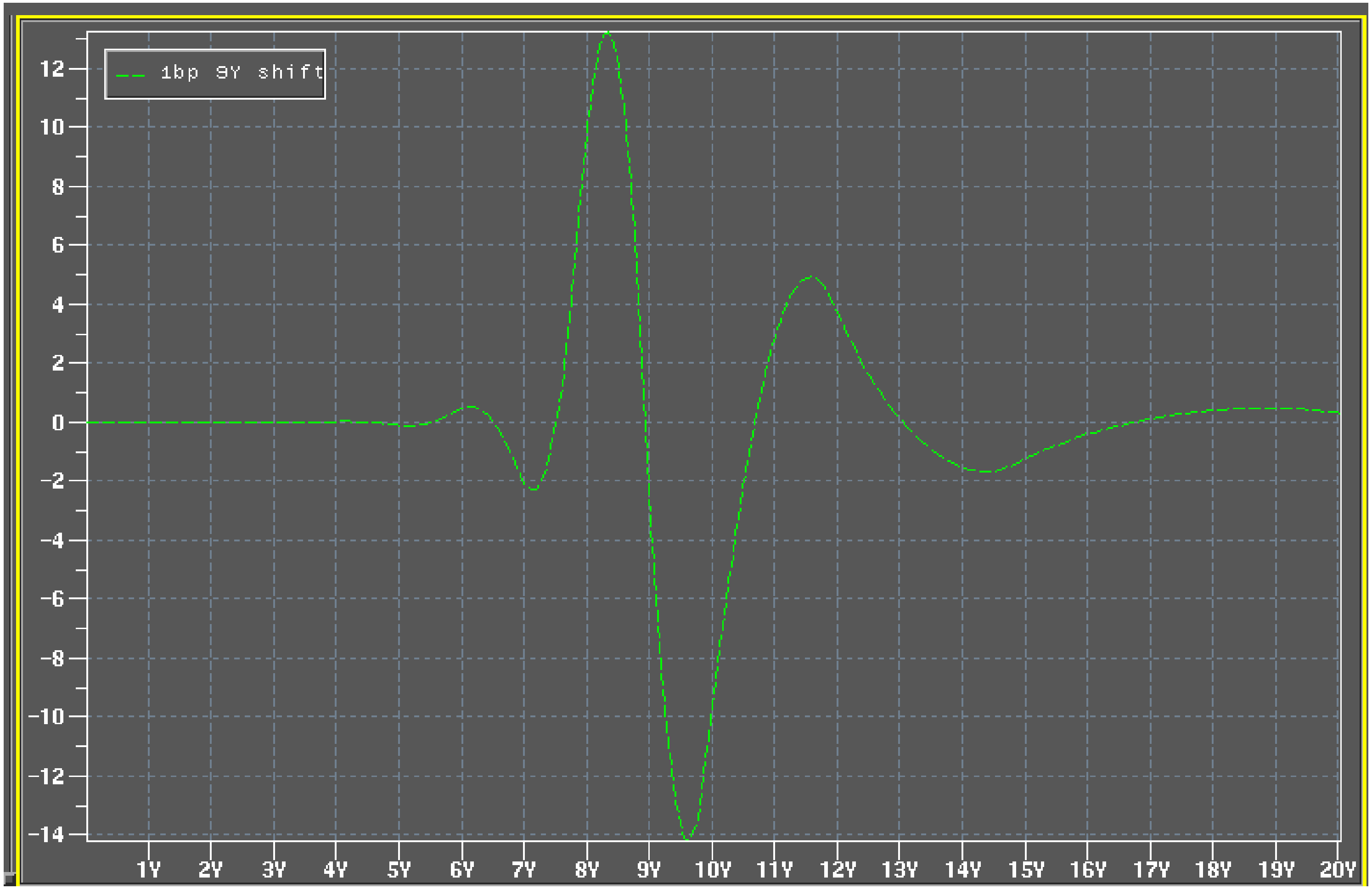
$$P = \sum_{i=1}^{N} e^{-st_i} \Lambda_i C_i$$

Where $\boldsymbol{\Lambda_i}$ - discount factor for payment date *i computed from the curve*
s – bond spread
t_i *– future time of payment, years*
Ci - i-th cashflow

- If model is available for typical movements of the curve embedded in Λ_i we can build more effective risk model for bond, rather than using single "parallel" shift mode (bond duration)

Shifting 9Y swap by 1 basis point

- Response of 3M forward rates



Portfolio Risk and Cost of Hedging

- Portfolio risk and Bid-Offer charge per bucket

Instrument	Quote	Raw Risk	B/O charge bp	Charge
IRS=1Y	0.33	(200,000)	0.10	20,000
IRS=2Y	0.39	1,330,000	0.10	133,000
IRS=3Y	0.49	(200,000)	0.25	50,000
IRS=4Y	0.64	1,200,000	0.25	300,000
IRS=5Y	0.86	(722,450)	0.10	72,245
IRS=6Y	1.09	(35,255)	0.25	8,814
IRS=7Y	1.29	(537,430)	0.25	134,358
IRS=8Y	1.48	(3,850,000)	0.25	962,500
IRS=9Y	1.64	1,580,000	0.25	395,000
IRS=10Y	1.79	288,751	0.10	28,875
IRS=12Y	2.04	(401,350)	0.25	100,338
IRS=15Y	2.29	50,000	0.25	12,500
IRS=20Y	2.50	4,000,000	0.25	1,000,000
IRS=25Y	2.60	(1,000,000)	0.25	250,000
IRS=30Y	2.67	(1,500,000)	0.10	150,000
TOTAL		2,266		3,617,629

Hedging Portfolio risks - Formulation

$$\mathbf{x} = \arg\min\{\| \mathbf{F}^{\mathbf{T}}(\mathbf{r} + \mathbf{H}\mathbf{x}) \|^{\mathbf{2}} \}$$

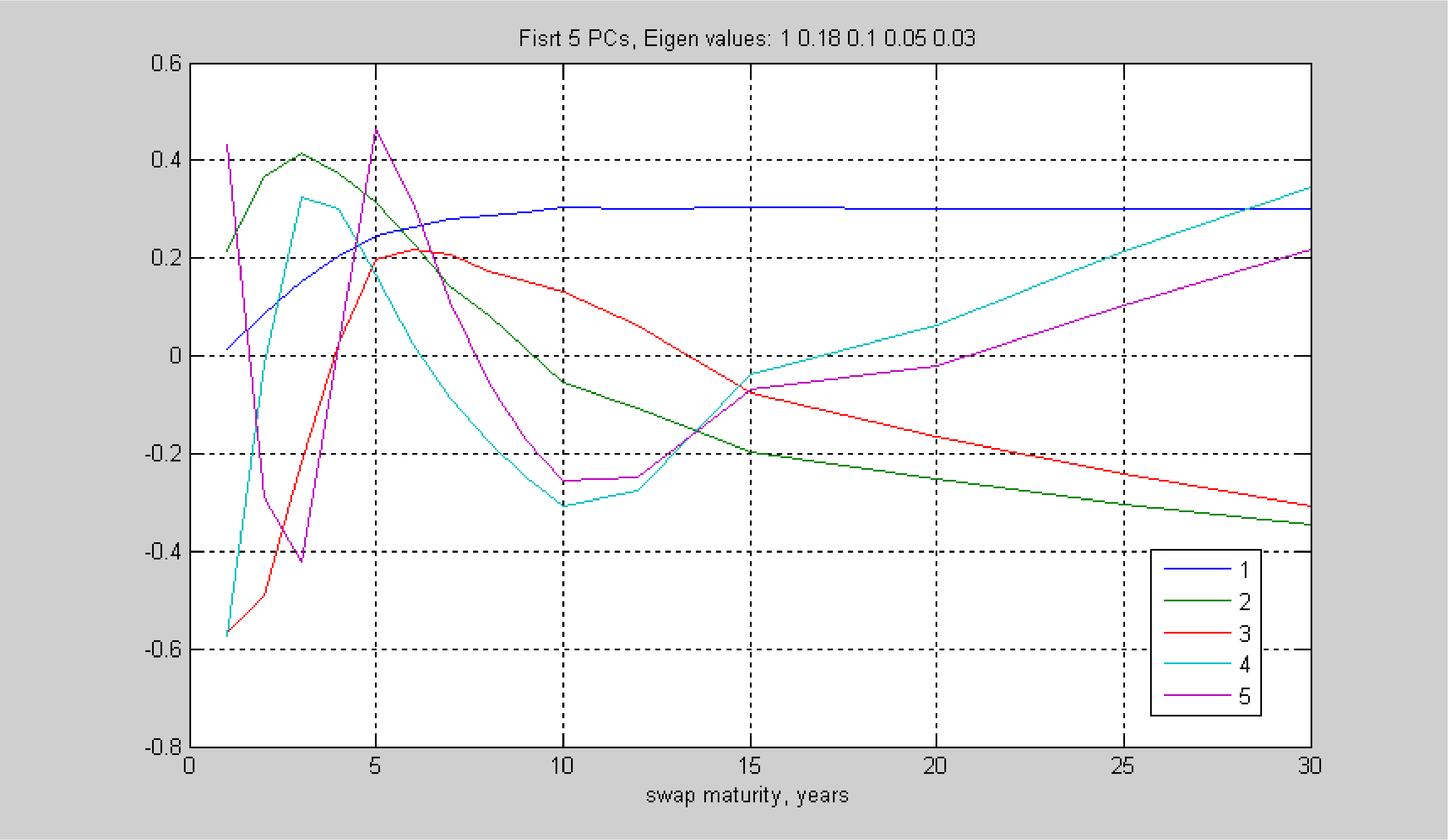
- **r** – portfolio risk
- **H** – hedging portfolio risks
- **x** – weights of hedging instruments
- **F** – market scenarios (factors)

Principal Component Analysis (PCA)

$$\mathbf{D} = \mathbf{U}\mathbf{S}\mathbf{P}^{\mathbf{T}}$$

- Use SVD to decompose market movements data **D** into principal components **P** and corresponding uncorrelated market dynamics **U** with weights **S**
- Use few SVD components with largest singular values - low rank approximation of market data
- Principal components **P** are eigen vectors of covariance matrix $\mathbf{D}^{\mathbf{T}}\mathbf{D}$

Main Principal Components of Swap Rates



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Hedging Portfolio Risks - PCA

$$\mathbf{P}^T(\mathbf{r} + \mathbf{H}\mathbf{x}) = \mathbf{0}$$

$$\mathbf{x} = (\mathbf{P}^T\mathbf{H})^{-1}\mathbf{P}^T\mathbf{r}$$

$$\mathbf{x} = \mathbf{R}^T\mathbf{r}$$

$$\mathbf{R} = \mathbf{P}(\mathbf{H}^T\mathbf{P})^{-1}$$

Hedging matrix **H**

Swap	1Y	2Y	5Y	10Y	30Y
IRS=1Y	1	0	0	0	0
IRS=2Y	0	1	0	0	0
IRS=3Y	0	0	0	0	0
IRS=4Y	0	0	0	0	0
IRS=5Y	0	0	1	0	0
IRS=6Y	0	0	0	0	0
IRS=7Y	0	0	0	0	0
IRS=8Y	0	0	0	0	0
IRS=9Y	0	0	0	0	0
IRS=10Y	0	0	0	1	0
IRS=12Y	0	0	0	0	0
IRS=15Y	0	0	0	0	0
IRS=20Y	0	0	0	0	0
IRS=25Y	0	0	0	0	0
IRS=30Y	0	0	0	0	1

- **P** – PCA factors
- **H** – risk of hedging portfolio (liquid swaps)
- **R** – risk transform matrix

Hedging using PCA model

		PCA Matrix				
Swap	Raw Risk	1Y	2Y	5Y	10Y	30Y
IRS=1Y	(200,000)	1.00	0.00	0.00	0.00	0.00
IRS=2Y	1,330,000	0.00	1.00	0.00	0.00	0.00
IRS=3Y	(200,000)	-0.51	1.16	0.29	-0.02	-0.02
IRS=4Y	1,200,000	-0.32	0.60	0.70	-0.04	-0.01
IRS=5Y	(722,450)	0.00	0.00	1.00	0.00	0.00
IRS=6Y	(35,255)	0.02	-0.05	0.80	0.28	-0.04
IRS=7Y	(537,430)	-0.01	-0.03	0.54	0.54	-0.04
IRS=8Y	(3,850,000)	-0.01	0.02	0.33	0.73	-0.05
IRS=9Y	1,580,000	-0.01	0.01	0.15	0.88	-0.03
IRS=10Y	288,751	0.00	0.00	0.00	1.00	0.00
IRS=12Y	(401,350)	0.02	0.01	-0.08	0.95	0.11
IRS=15Y	50,000	0.01	0.00	-0.04	0.59	0.45
IRS=20Y	4,000,000	0.01	0.03	-0.08	0.44	0.62
IRS=25Y	(1,000,000)	0.00	0.02	-0.04	0.20	0.82
IRS=30Y	(1,500,000)	0.00	0.00	0.00	0.00	1.00
	PCA risk	**(426,757)**	**1,892,770**	**(1,538,808)**	**(268,764)**	**293,261**
	B/O charge	0.1	0.1	0.1	0.1	0.1
	Charge	**42,676**	**189,277**	**153,881**	**26,876**	**29,326**

PCA Risk Model

- “Formally” tuned to historical data
- Hedge coefficients are not stable, especially if historical widow is short to reflect recent regime
- Costly to re-hedge when PC factors change
- Instability is coming from noisy PCs corresponding to small singular values
- Over-fitting to historical data
- No assumptions used about shape of the yield curve

PCA Interpretation

- Risk matrix **R** is linear combinations **Y** of principal components **P** producing shifts of one hedging instrument at a time

$$\mathbf{R} = \mathbf{P}\mathbf{Y}$$

$$\mathbf{H}^{T}\mathbf{R} = \mathbf{I}$$

$$\mathbf{R} = \mathbf{P}(\mathbf{H}^{T}\mathbf{P})^{-1}$$

- Can we build risk model **R** based on some reasonable assumption (such as smoothness of forward rates) rather than purely historical data?

Regularized Risk Model

- Assumption: Forward rates move smoothly

$$\mathbf{H}^{\mathbf{T}}\mathbf{R} = \mathbf{I}$$

$$\| \mathbf{LJR} \|^2 \to \min$$

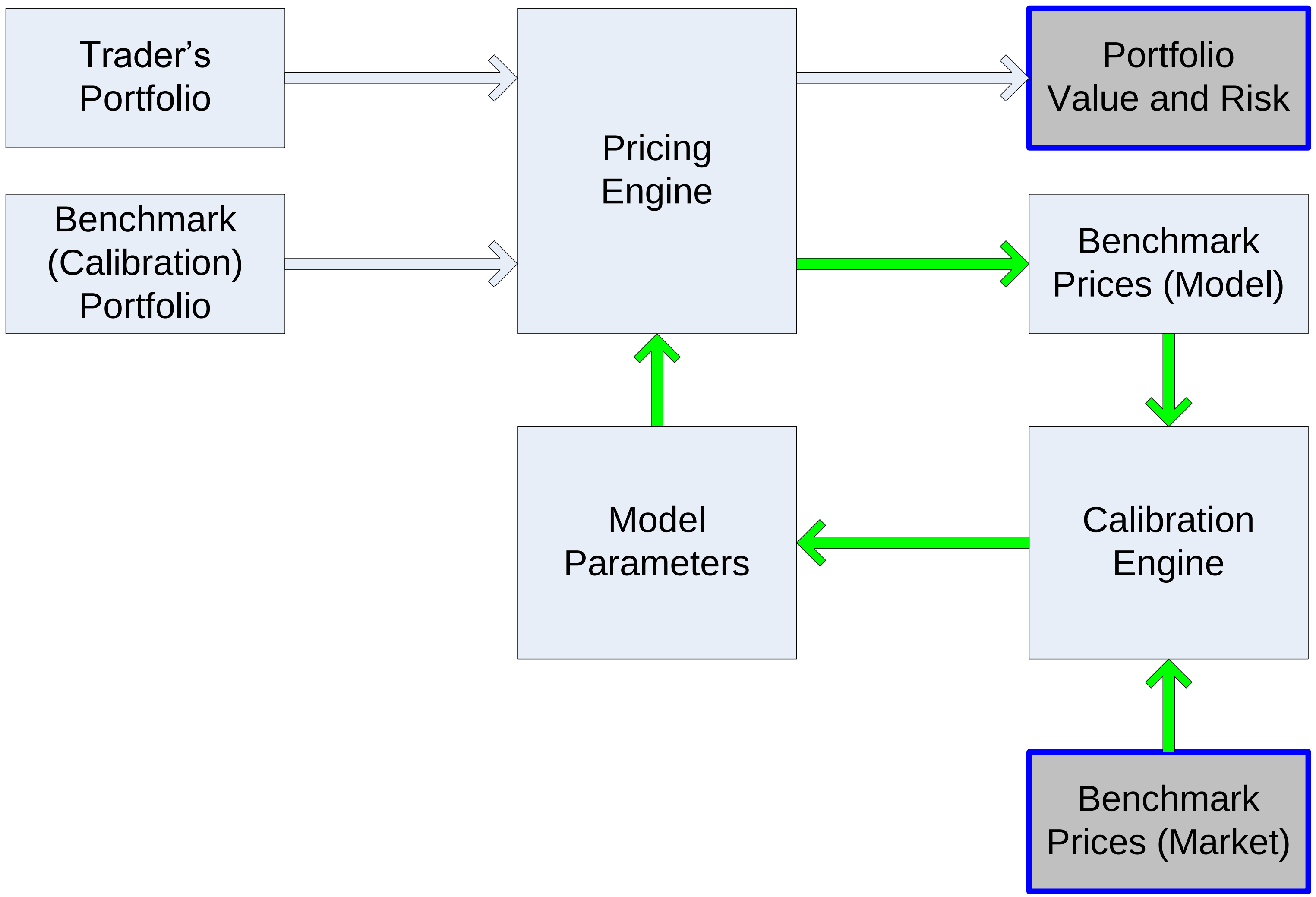
$$\mathbf{R} \sim (\mathbf{HH}^{\mathbf{T}} + \boldsymbol{\lambda}^{\mathbf{2}} (\mathbf{LJ})^{\mathbf{T}} \mathbf{LJ})^{-1}$$

- Where J – Jacobian matrix translating shifts of yield curve inputs to movements of forward rates, L – smoothness regularity matrix, λ- small regularization parameter

Regularized Model Risk Projection

Swap	Raw Risk	1Y	2Y	5Y	10Y	30Y
IRS=1Y	(200,000)	1.00	0.00	0.00	0.00	0.00
IRS=2Y	1,330,000	0.00	1.00	0.00	0.00	0.00
IRS=3Y	(200,000)	-0.28	0.95	0.44	-0.12	0.01
IRS=4Y	1,200,000	-0.13	0.36	0.92	-0.17	0.01
IRS=5Y	(722,450)	0.00	0.00	1.00	0.00	0.00
IRS=6Y	(35,255)	0.04	-0.11	0.81	0.28	-0.02
IRS=7Y	(537,430)	0.04	-0.12	0.58	0.53	-0.04
IRS=8Y	(3,850,000)	0.03	-0.09	0.35	0.75	-0.04
IRS=9Y	1,580,000	0.02	-0.04	0.15	0.90	-0.03
IRS=10Y	288,751	0.00	0.00	0.00	1.00	0.00
IRS=12Y	(401,350)	-0.02	0.05	-0.17	1.05	0.09
IRS=15Y	50,000	-0.03	0.07	-0.24	0.93	0.27
IRS=20Y	4,000,000	-0.02	0.05	-0.19	0.59	0.56
IRS=25Y	(1,000,000)	-0.01	0.03	-0.09	0.26	0.81
IRS=30Y	(1,500,000)	0.00	0.00	0.00	0.00	1.00
TOTAL	2266	**(487,769)**	**2,082,997**	**(1,752,958)**	**78,962**	**64,483**

Pricing Model Diagram



Heath-Jarrow-Morton (HJM) Model

- Evolution of forward rates

$$df_{t,s} = \mu_{t,s} dt + f_{t,s}^{\beta} V(t,s)\rho(t,s) \cdot dB_t^Q$$

f – forward rate

μ- drift

β – model skew factor

ρ – correlation/factor structure

V(t,s) – parametric volatility surface (our main focus today)

dB_t^Q - Brownian motion

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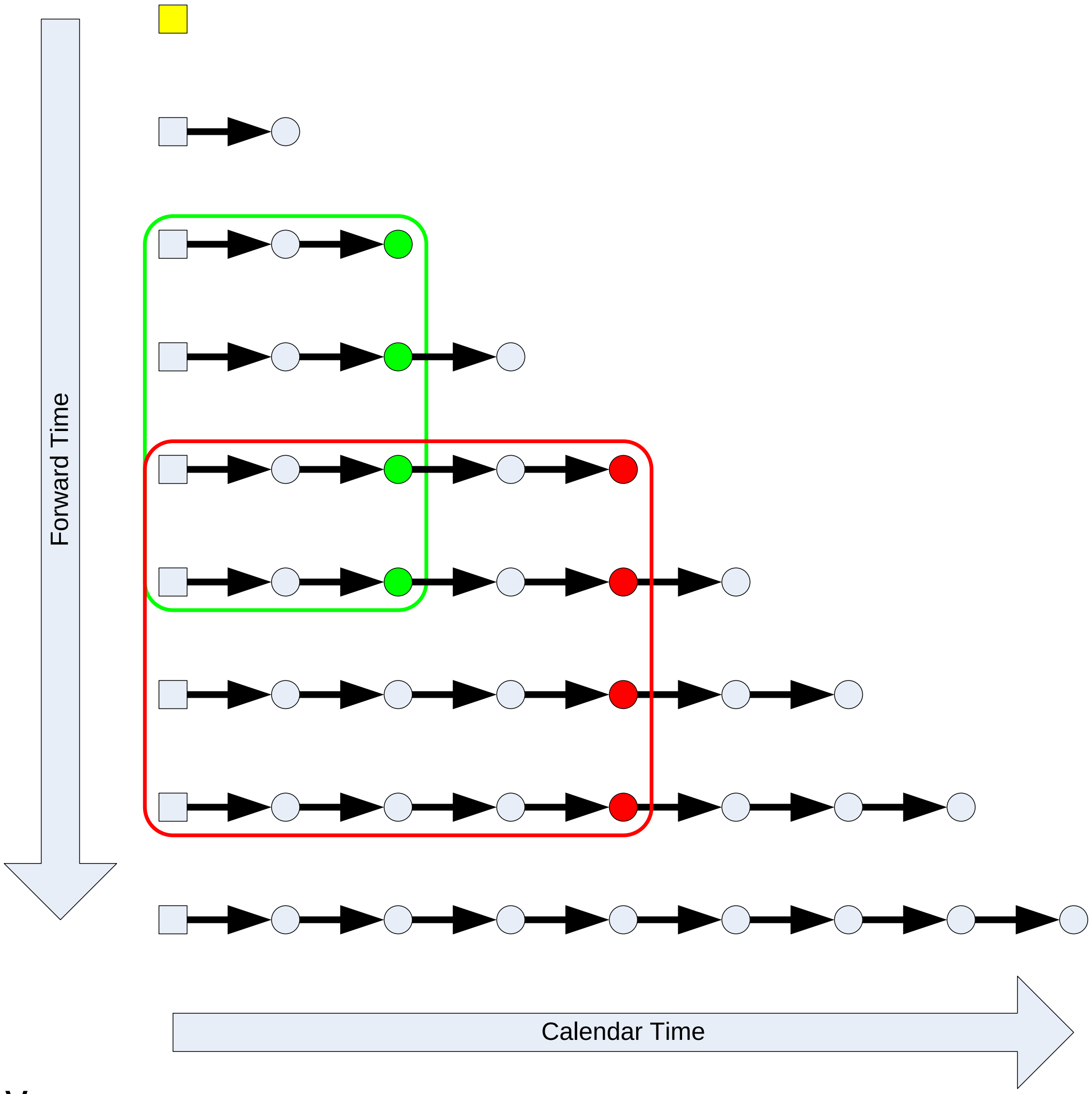
Forward Rates Map



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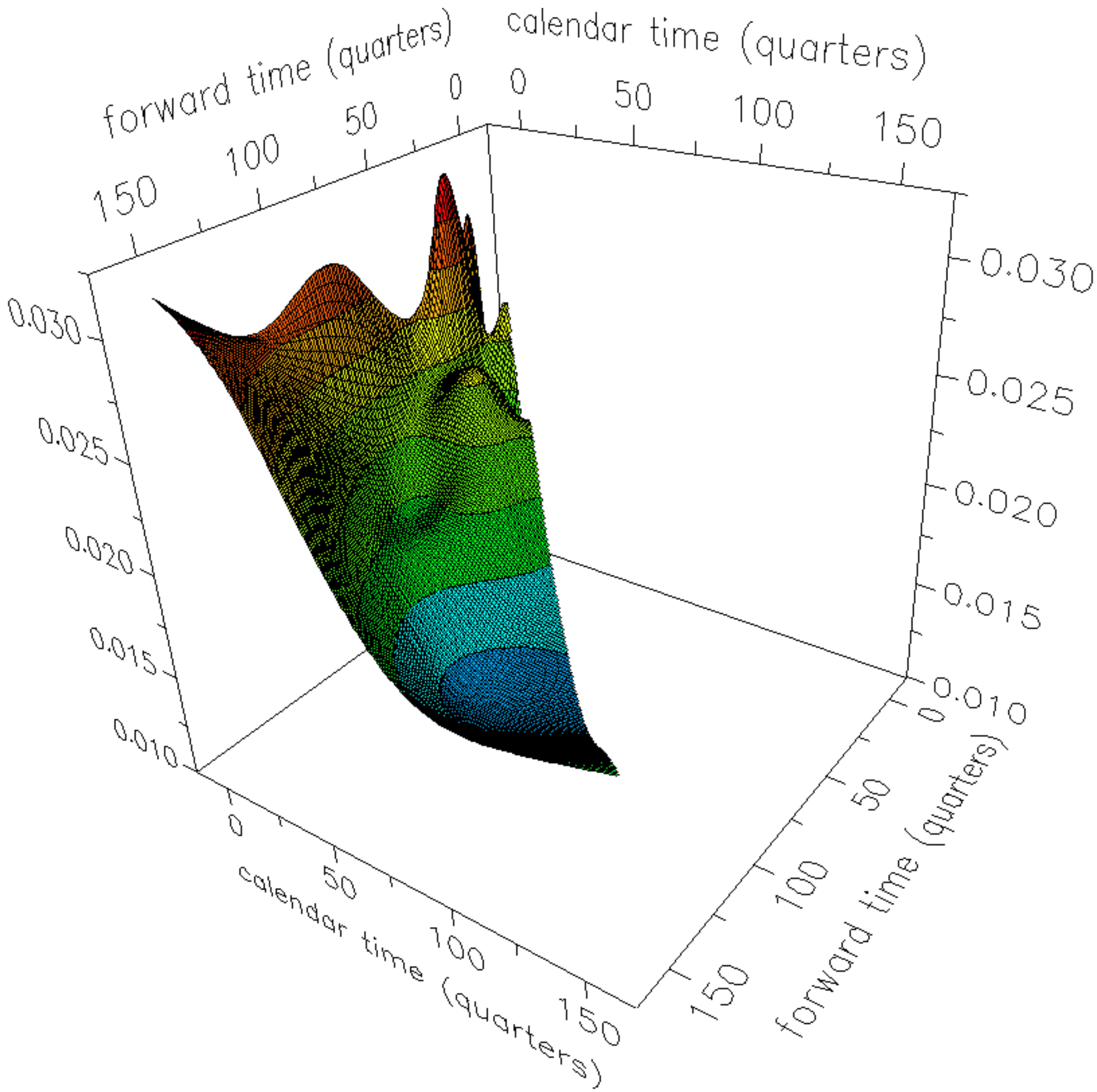
Parametric Volatility Surface



Volatility Surface Calibration Challenge

- High dimensionality (need to calibrate ~28k elements)
- No memory to store 28k X 28k matrix
- Relatively small number of calibration instruments (20-50)
- Under-determined problem
- Sensitivity areas of calibration instruments overlap significantly
- Ill posed inverse problem
- Unstable, noisy solution
- Need regularity constraints
- Has to be smooth to produce realistic prices for similar instruments

Formal Approach to Calibration

- Represent volatility surface as a linear combination of N basis functions

$$\mathbf{v} = \mathbf{v_0} + \mathbf{B} \cdot \mathbf{x}$$

v – vector containing elements of the volatility grid

B – matrix, columns corresponding to basis fuctions

x – vector of weights

- Make N equivalent to the number of calibration instruments M
- "Formally" unambiguous
- Make basis functions piecewise constant matching sensitivity of calibration instruments, 0 otherwise

Compute sensitivities (Jacobian matrix)

- Use pricing model to compute sensitivities of prices of calibration instruments to perturbations of volatility surface

$$J_{ij} = \frac{\partial q_i}{\partial x_j}$$

$$\mathbf{q} = \mathbf{J} \cdot \mathbf{x}$$

$$\mathbf{q} = \ln \frac{\mathbf{q_{mdl}}}{\mathbf{q_0}}, \mathbf{q_{in}} = \ln \frac{\mathbf{q_{market}}}{\mathbf{q_0}}$$

Where **J** – Jacobian matrix

$\mathbf{q_{mdl}}$, $\mathbf{q_{market}}$, $\mathbf{q_0}$ – model, market, and base price

x – vector of basis functions coefficients

Solve

- **J** is square and invertible, as basis functions are selected reasonably
- Iteratively solve for basis function coefficients **x**

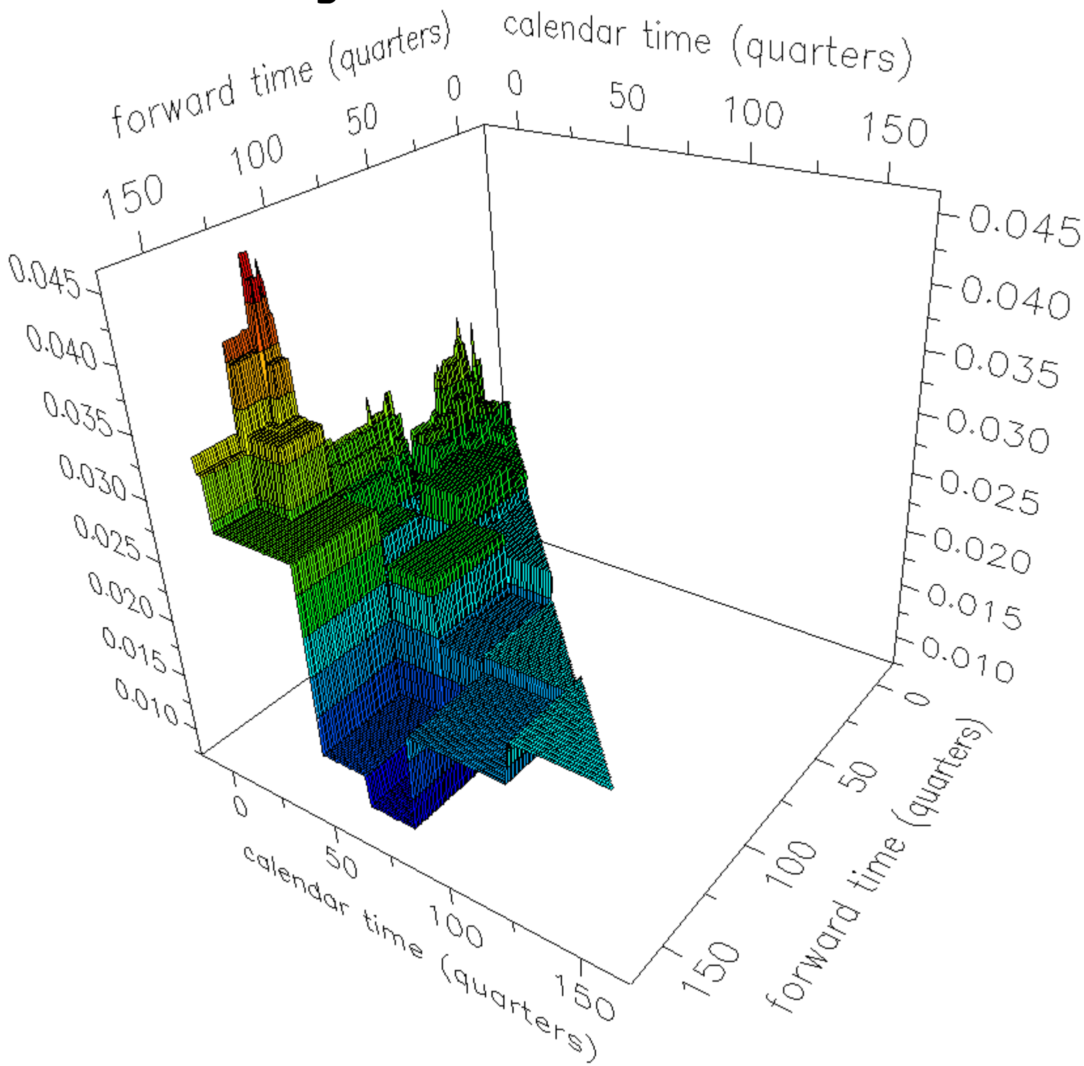
$$\mathbf{q_{in}} = \mathbf{J} \cdot \mathbf{x}$$

$$\mathbf{x} = \mathbf{J}^{-1} \mathbf{q_{in}}$$

- Quickly converges, as (typically) price is ~proportional to volatility for at-the-money calibration instruments

“Formal” solution

- Exact, but … meaningless



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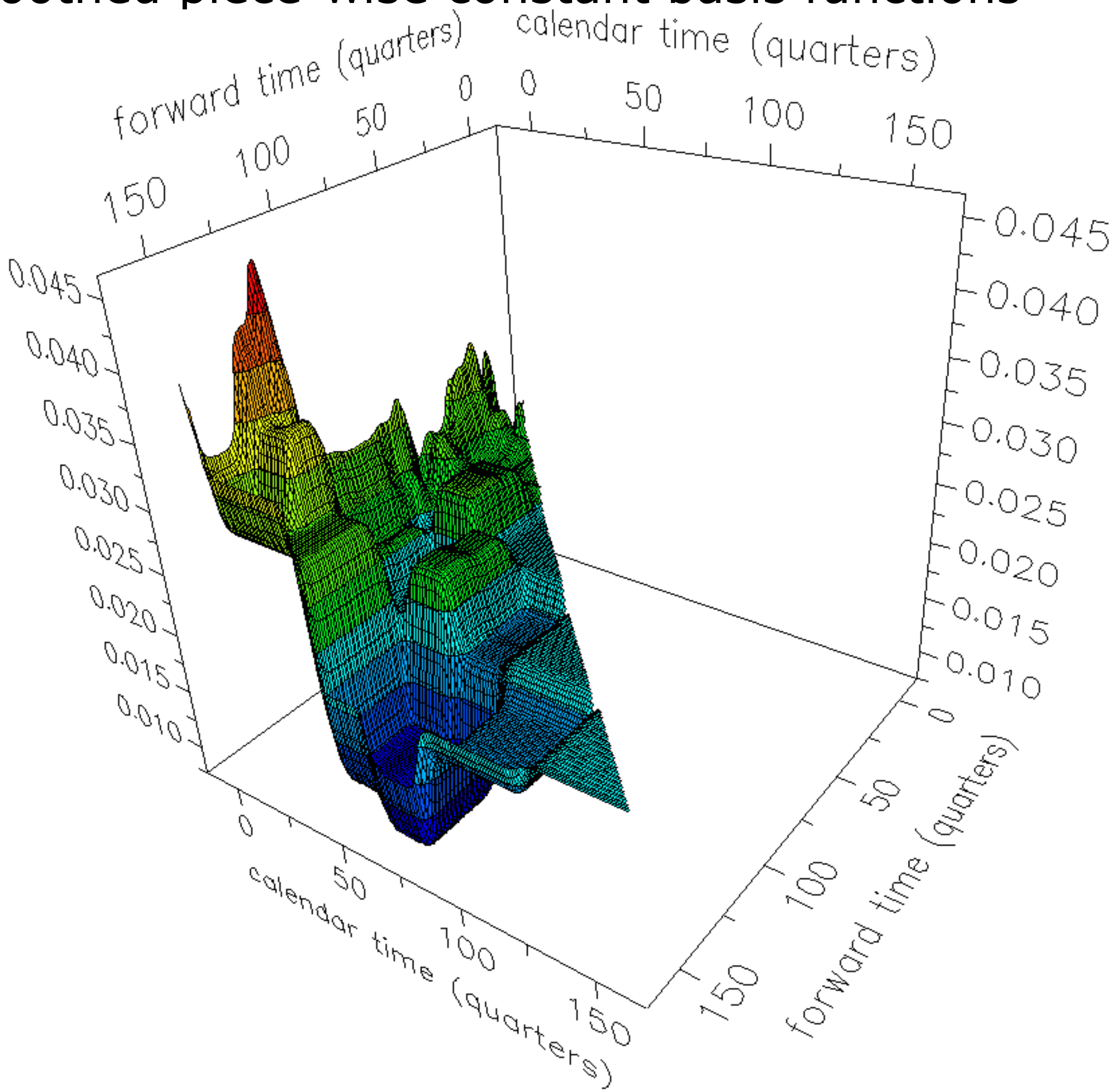
Attempt to improve solution

- Using smoothed piece-wise constant basis functions



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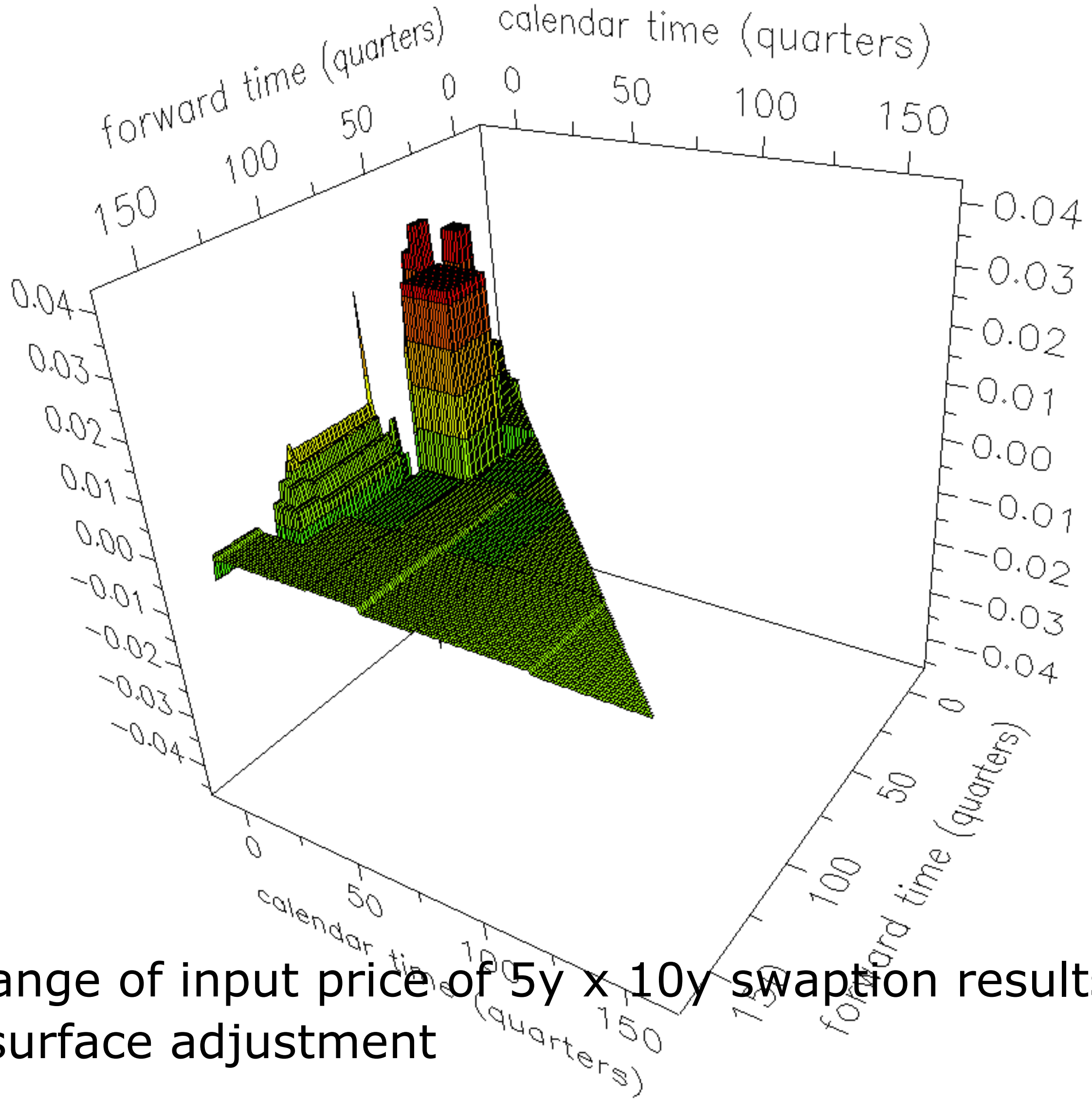
Calibration problem: Ill-posed

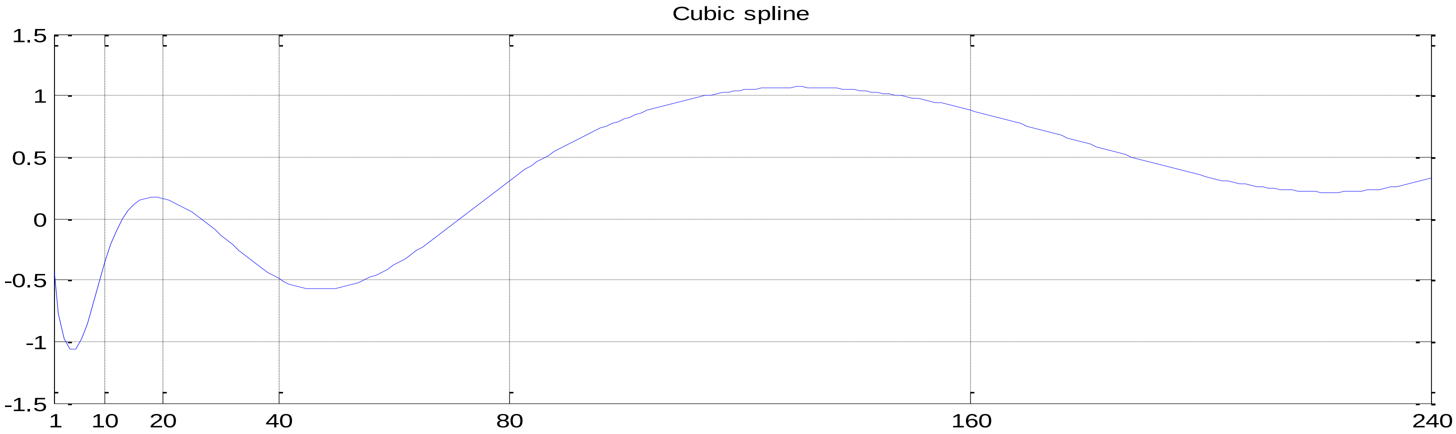


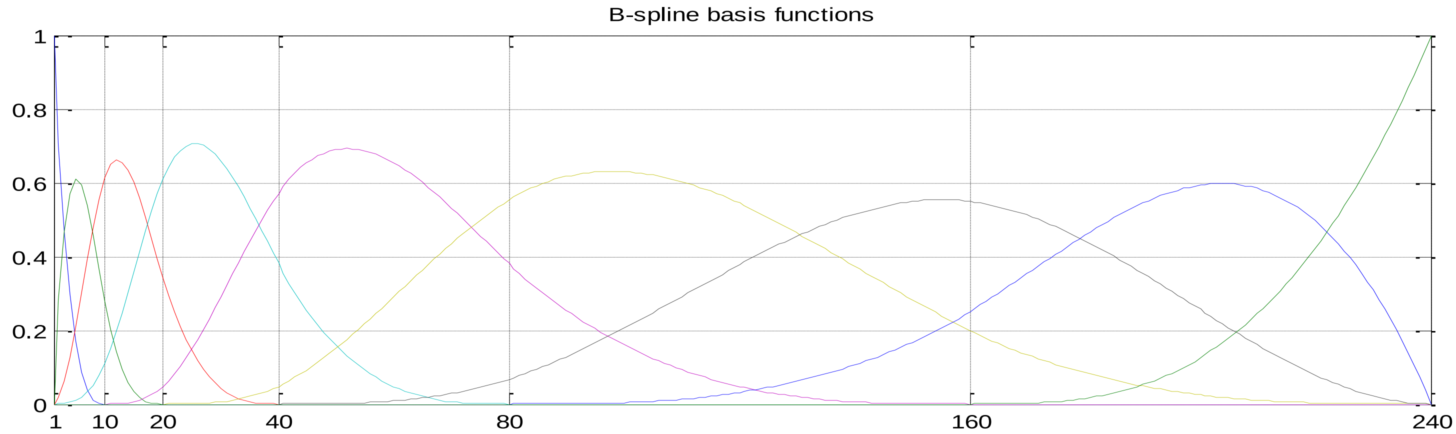
- 1% change of input price of 5y x 10y swaption results in 4% change of vol surface adjustment

Key Improvements of Calibration

- Use ill-posedness to our advantage:
  - Allow some tolerance to calibration accuracy of input instruments
  - Significant improvements in the (smoothness of) output surface may not cost much in terms of accuracy of calibration
  - Calibration instruments have different liquidity, and bid-offer spread. So we can use weights to decrease tolerance for important instruments
- Basis functions:
  - Absolutely need basis functions to reduce dimensionality of the inverse problem
  - Need many (M > N instruments) basis functions, as we do not know in advance which shapes will work
  - 2-dimensional B-Splines

B-Spline representation





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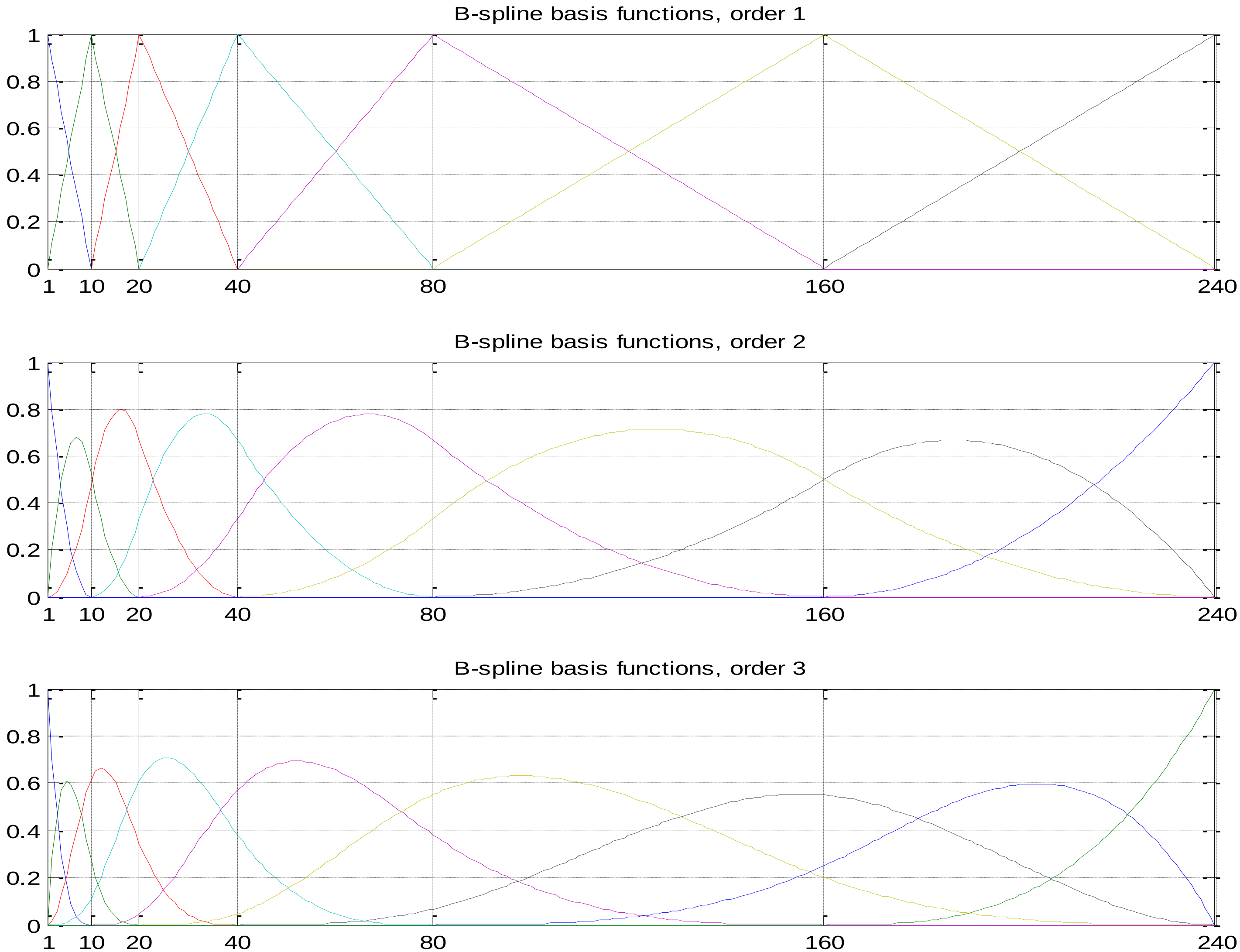
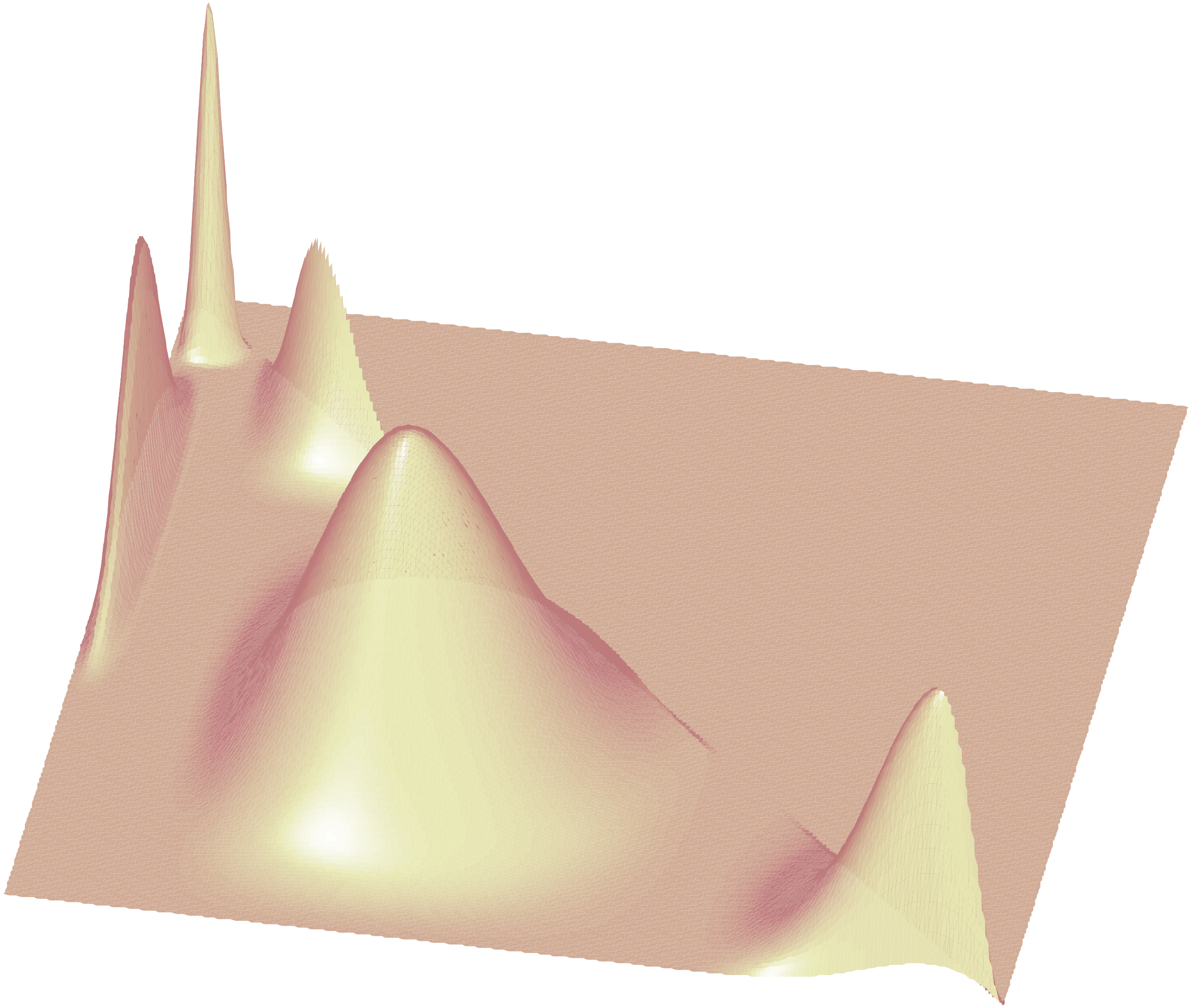
Building B-splines using Cox-de Boor recursion formula



2-d B-spline functions



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Formulate the problem

$$\| \mathbf{W} \cdot (\mathbf{q} - \mathbf{q_{in}}) \|^2 \rightarrow \min$$

$$\| \mathbf{L_1} \cdot (\mathbf{v} - \mathbf{v_0}) \|^2 \rightarrow \min$$

$$\| \mathbf{L}_2 \cdot \mathbf{v} \|^2 \rightarrow \min$$

Where **W** – diagonal matrix of weights
$\mathbf{L_1}$ – regularization matrix for change
$\mathbf{L_2}$ – regularization matrix for result

Surface Gradient Penalty

Example of regularization matrix $\mathbf{L_1}$

$$
\mathbf{L_1} =
\begin{matrix}
1 & -1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 1 & 0 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \ldots & -1 & 0 & 0
\end{matrix}
$$

Solution of Regularized Optimization Problem

$$\mathbf{x} = \arg\min\{\| \mathbf{W}(\mathbf{Jx} - \mathbf{q_{in}}) \|^2 + \| \lambda_1 \mathbf{L_1 Bx} \|^2 + \| \lambda_2 \mathbf{L_2 x} \|^2\}$$

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{J^T W^2 q_{in}}, \textit{where}$$

$$\mathbf{A} = (\mathbf{J^T W^2 J} + \lambda_1^2 (\mathbf{L_1 B})^{\mathbf{T}} \mathbf{L_1 B} + \lambda_2^2 \mathbf{L_2^T L_2})^{-1}$$

Where $\mathbf{L_2}$ – Tikhonov regularization matrix

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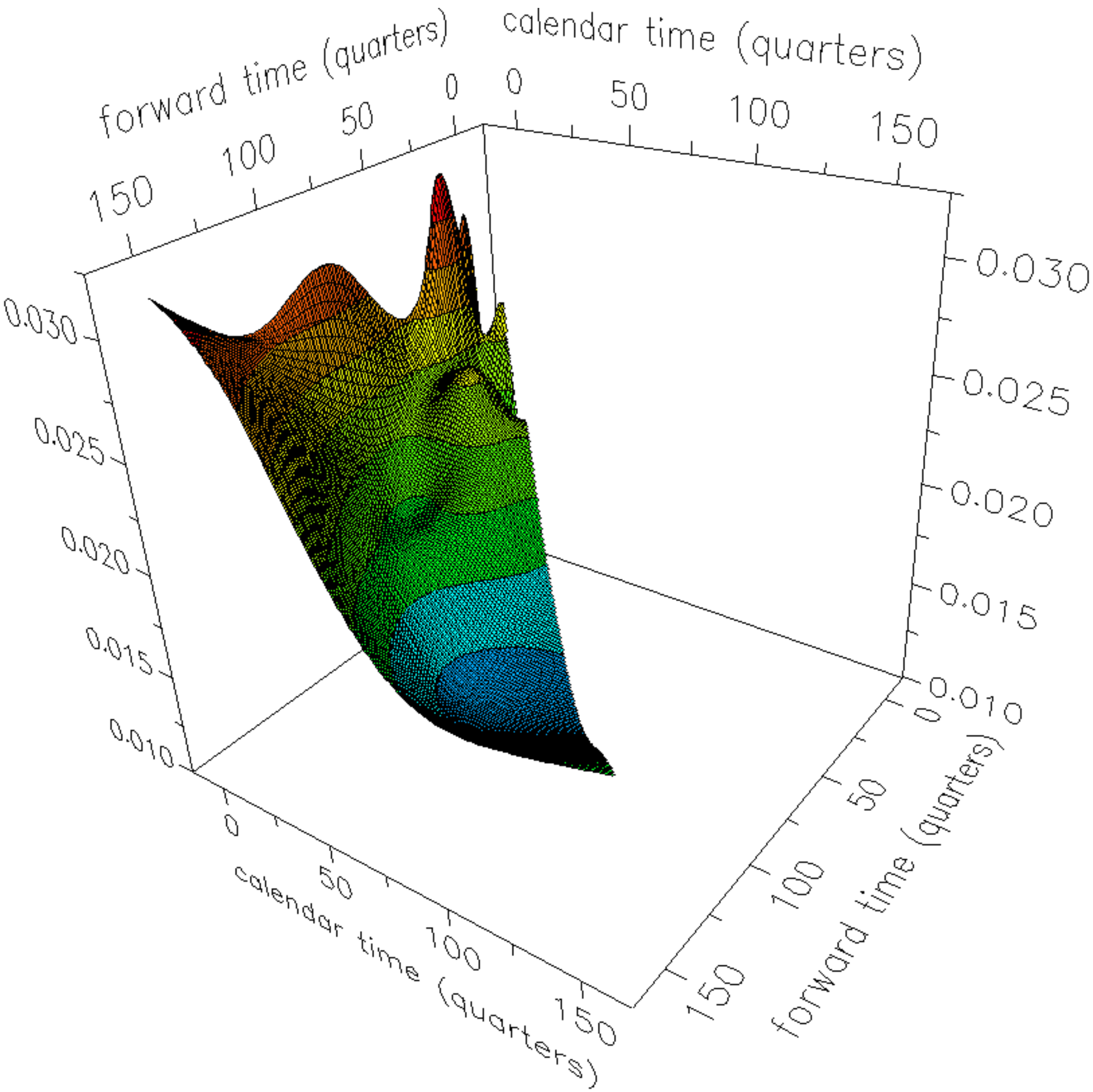
Calibration Result



Calibration Inverse Problem

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \varepsilon$$

$$\mathbf{x} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{y}$$

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} = \sum_i s_i \mathbf{u_i}\mathbf{v_i^T}$$

$$\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{I}$$

$$\mathbf{x} = \mathbf{V}\mathbf{S}^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{y} = \sum_i \frac{\mathbf{u_i^T}\mathbf{y}}{s_i}\mathbf{v_i}$$

- **y** – market inputs, **x** – model parameters
- Singular Value Decomposition of (forward) model matrix **A**
- s_i – singular values
- Result: Rotation → Scaling $1/s_i$ → Rotation

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Ill Posed Problem

$$\mathbf{x} = \sum_i \frac{\mathbf{u}_\mathbf{i}^\mathbf{T} \boldsymbol{\varepsilon}}{s_i} \mathbf{v}_\mathbf{i}$$

- Input noise $\boldsymbol{\varepsilon}$ may be magnified by small singular values s_i
- Condition number $\max(s_i)/\min(s_i)$ as indicator of ill-posedness
- Small variation in input results in large change in the solution

"Noiseless" situation

$$\mathbf{x} = \sum_i \frac{\mathbf{u_i^T y}}{s_i} \mathbf{v_i}$$

$$\mathbf{p} = \mathbf{Bx}$$

$$\mathbf{p_y} = \mathbf{Ax} = \mathbf{A(A^T A)^{-1} A^T y} = \mathbf{y}$$

- We compute **x** to calculate price of the portfolio **p**
- If all singular values are non-zero, we "formally" re-price inputs, as $A^T(A^TA)^{-1}A=I$ – identity. So, the model appears to be accurate.
- However, pricing of actual portfolio **p** with model **B** may be unstable

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Tikhonov Regularization

$$\mathbf{x} = \arg\min\{\| \mathbf{Ax} - \mathbf{y} \|^2 + \| \lambda \mathbf{x} \|^2\}$$

$$\mathbf{x} = (\mathbf{A}^{\mathbf{T}}\mathbf{A} + \boldsymbol{\lambda}^{\mathbf{2}}\mathbf{I})^{-1}\mathbf{A}^{\mathbf{T}}\mathbf{y}$$

$$\mathbf{x} = \mathbf{V}\frac{\mathbf{S}}{\mathbf{S}^{\mathbf{2}} + \lambda^2\mathbf{I}}\mathbf{U}^{\mathbf{T}}\mathbf{y} = \sum_i w_i \cdot \frac{\mathbf{u}_{\mathbf{i}}^{\mathbf{T}}\mathbf{y}}{s_i}\mathbf{v}_{\mathbf{i}}$$

$$w_i = \frac{s_i^2}{s_i^2 + \lambda^2}$$

- Apply penalty to amplitude of model parameters **x**
- Re-pricing matrix $A^T(A^TA+\lambda^2I)^{-1}A \neq I$ is no longer 100% accurate, however
- More stable model vector **x**, and pricing of actual portfolio

Truncated SVD (TSVD)

$$\mathbf{x} = \sum_i w_i \cdot \frac{\mathbf{u_i^T y}}{s_i} \mathbf{v_i}$$

$$w_i = 1, i \leq N$$

$$w_i = 0, i > N$$

- Truncation of effective rank of the model matrix **A**
- Similarity with PCA (principal component) approach
- Truncated is “null” space of the model: parameter modes, which do not affect calibration accuracy
- The problem is when “null” space of the model has noticeable impact on portfolio pricing

Regularized Models

- Improved stability
- Regularization is essential for ill-conditioned problems
- More realistic solution at the expense of fitting input data
- May cause bias to the solution
- Bias can be minimized by proper selection of the penalty constraints

Useful Links

- HJM model: http://en.wikipedia.org/wiki/HJM_model
- Yield Curve: http://en.wikipedia.org/wiki/Yield_curve
- Inverse problems: http://en.wikipedia.org/wiki/Inverse_problem
- Tikhonov Regularization:
  http://en.wikipedia.org/wiki/Tikhonov_regularization
- Singular Value decomposition:

  http://en.wikipedia.org/wiki/Singular_value_decomposition
- PCA: http://en.wikipedia.org/wiki/Principal_component_analysis
- B-Splines: http://en.wikipedia.org/wiki/B-spline

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