# 18.S096 Problem Set 9 Fall 2013

## Stochastic Differential Equations

Due date **: 12/5/13 (no need to turn in)**
Collaboration on homework is encouraged, but you should think through the problems yourself before discussing them with other people. **You must write your solution in your own words. Make sure to list all your collaborators.**

## Part A

Part A has problems that straightforwardly follow from the definition. Use this part as an opportunity to get used to the concepts and definitions.

**Problem A-1**. Verify that the given processes solve the given corresponding stochastic differential equations.
(a) $X_t = exp(B_t)$ solves

$$dX_t = \frac{1}{2}X_t dt + X_t dB_t.$$

(b) $X_t = \frac{B_t}{1+t}$ solves

$$dX_t = -\frac{1}{1+t}X_t dt + \frac{1}{1+t}dB_t.$$

(c) $X_t = \sin B_t$ solves

$$dX_t = -\frac{1}{2}X_t dt + \sqrt{1-X_t^2}dB_t.$$

**Problem A-2.** Let $a > 0$ and suppose that

$$dX_t = \frac{1}{3}X_t^{1/3}dt + X_t^{2/3}dB_t.$$

Show that

$$X_t = (a^{1/3} + \frac{1}{3}B_t)^3 \quad t \geq 0,$$

solves the SDE given above when the initial condition is given as $X_0 = a$.

**Problem A-3.** (Vasicek interest rate model) Prove that

$$R(t) = e^{-\beta t}R(0) + \frac{\alpha}{\beta}(1 - e^{-\beta t}) + \sigma e^{-\beta t}\int_0^t e^{\beta s}dB_s$$

solves the SDE

$$dR(t) = (\alpha - \beta R(t))dt + \sigma dB_t.$$

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