Massachusetts Institute of Technology
18.965 Fall 2004
Homework 5

Due Friday 12/10/04

Exercise 1. Let

$$f(z_0, z_1, z_2, \ldots, z_n) = \frac{\sum_{i=0}^{n}(i+1)|z_i|^2}{\sum_{i=0}^{n}|z_i|^2}$$

This defines a function on the projective space $\mathbb{CP}^n$. Find the critical points of this function and compute their indices. Use the Morse complex to compute the homology of $\mathbb{CP}^n$.

Exercise 2. Find a good cover of a surface of genus g and compute the Čech cohomology of this cover.

Exercise 3. Prove the following formulae where α and β are forms and v and w are vector fields.

1. $\iota_v(\alpha \wedge \beta) = (\iota_v \alpha) \wedge \beta + (-1)^{|\alpha|}\alpha \wedge (\iota_v \beta)$
2. $[\mathcal{L}_v, \mathcal{L}_w] = \mathcal{L}_{[v,w]}$

Exercise 4. Find a formula for $\mathcal{L}_v \alpha$ in local coordinates.

Exercise 5. Show that if $\mathfrak{G}$ is a good cover (all intersections are contractible or empty) and $\mathfrak{U}$ is a refinement of $\mathfrak{G}$ then

$$\check{H}(\mathfrak{U}) \equiv \check{H}(\mathfrak{G})$$