

18.965 Fall 2004  
Homework 5

Due Monday 11/8/04

*Exercise 1.* Let  $M$  be smooth manifold embedded in  $\mathbb{R}^N$ . Show that for a residual subset of the dual space of  $\mathbb{R}^N$  the restriction on a linear functional to  $M$  is a Morse function.

*Exercise 2.* Show that every Morse function on a closed orientable 2 dimensional manifold of genus  $g$  has at least  $2g + 2$  critical points.

*Exercise 3.* Show that the space of  $n \times n$  symmetric matrices with at least one eigenvalue of multiplicity greater than one is a stratified space with the stratum stratum of largest dimension being of codimension three.

*Exercise 4.* Show that every real vector bundle  $\xi$  of rank  $k$  over a manifold  $X$  of dimension  $d$  can be pulled back from the grassmanian

$$Gr_k(\mathbb{R}^{k+d}).$$

Hint: If  $\xi$  is pulled back then there is a fiberwise injective map

$$\xi \rightarrow \epsilon^{k+d}$$

Use parameteric transeversality to find the codimension of the set where a fiberwise linear map

$$\xi \rightarrow \epsilon^{k+d}$$

is not of full rank.