18.950 Differential Geometry
Fall 2008

18.950 Homework 5

1. (10 points) Let f be a hypersurface patch. Suppose that f lies in the half-plane $\{y_{n+1} \geq 0\} \subset \mathbb{R}^{n+1}$, and that f is tangent to the hyperplane $\{y_{n+1} = 0\}$ at $x = 0$. Prove that then, the principal curvatures at $x = 0$ satisfy $\lambda_i \lambda_j \geq 0$ for all i, j.

2. (3 points) Let f be a hypersurface patch of the form $f(x) = (x, \phi(x))$ for some $\phi : U \to \mathbb{R}$. Suppose that at the origin $x = 0$, both ϕ and $D\phi$ vanish. Compute the Christoffel symbols and their (first order) derivatives at that point.

3. (7 points) Let $f : U \to \mathbb{R}^3$ be a surface patch. Define the parallel surface at distance ϵ to be

$$\tilde{f}(s,t) = f(s,t) + \epsilon \cdot \nu(s,t),$$

where ν is the Gauss normal vector. Show that the principal curvatures of f and $\tilde{f}$ are related by $\tilde{\lambda}_i = \lambda_i/(1 - \epsilon\lambda_i)$ $(i = 1, 2)$. You may assume that ϵ is as small as needed for the argument.