18.950 Differential Geometry
Fall 2008

18.950 Homework 4

Problem 1. Prove that $\Lambda^2(L)$ vanishes if and only if L has rank ≤ 1.

Problem 2. Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be an invertible linear map. Prove that with a suitable choice of basis of $\Lambda^2(\mathbb{R}^3)$, the map $\Lambda^2(L) : \Lambda^2(\mathbb{R}^3) \to \Lambda^2(\mathbb{R}^3)$ turns into $(L^{-1})^{tr}\det(L)$.

Problem 3. (2 points) Determine the first and second fundamental form, as well as the principal curvatures, of the cylinder $f : \mathbb{R}^2 \to \mathbb{R}^3$, $f(s,t) = (s, \cos(t), \sin(t))$.

Problem 4. (4 points) Suppose that f parametrizes a piece of the standard sphere, so that the unit normal vector at each point is $\nu(x) = -f(x)$. From this fact, deduce that all principal curvatures are equal to $+1$.

Problem 5. (6 points) Define the third fundamental form of a hypersurface at a point x to be

$$III_x(X,Y) = \langle KX, Y \rangle,$$

where the coefficients of K are $k_{ij}(x) = \langle \partial_{x_i}\nu, \partial_{x_j}\nu \rangle$. Prove that $III_x(X,Y) = I_x(L^2X,Y)$.