18.950 Differential Geometry
Fall 2008

18.950 Homework 3

Problem 1. (3 points) Write down explicitly a curve $c : [0, \infty) \to \mathbb{R}^2$ such that the curvature $\kappa(t)$ goes to infinity as $t \to \infty$.

Problem 2. (7 points) Let $c : \mathbb{R} \to \mathbb{R}^2$ be a closed curve of period 5. Suppose that it also satisfies

$$c(t+1) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} c(t),$$

where $\alpha = 2\pi/5$. What can one say about the rotation number of c?

Problem 3. (10 points) A polygonal curve is a map $c : I \to \mathbb{R}^2$ with the property that there are $t_1 < \cdots < t_m$ in I such that

$$c(t) = c_0 t + v_0 \text{ for } t \leq t_1,\ c(t) = c_1 + tv_1 \text{ for } t_1 \leq t \leq t_2, \ldots$$

Here $c_i \in \mathbb{R}^2$, and v_i are nonzero vectors in $\mathbb{R}^2$. Moreover, (v_i, v_{i+1}) should never point in opposite directions.

Define an appropriate notion of curvature for a polygonal curve, and of total curvature for a closed polygonal curve (of course, defining closed polygonal curves first!). Does the Hopf Umlaufsatz still hold? Is there a version of Proposition 6.3 from the class? (for the last two questions, answers with sketch proofs are enough).