18.950 Differential Geometry
Fall 2008

**18.950 Homework 10**

**1.** (6 points) Check the formula for the geodesic equations on surfaces of rotation from lecture 32.

**2.** (6 points) As before, consider a surface of rotation. Given $c : I \to \mathbb{R}^2$, define the angular momentum to be $\tau = l_1(c)^2 c_2'$. Prove that if $\gamma = f(c)$ is a geodesic, then $\tau$ is constant.

Now consider the case of the hyperboloid created by rotating the curve $\{x_1^2 = 1 + x_2^2\}$ in the plane. In the following, we consider only geodesics which have unit speed. Prove that a geodesic with angular momentum $< 1$ goes from one end of the hyperboloid to the other, while one with angular momentum $> 1$ is confined to one half of the hyperboloid.

**3.** (8 points) In $\mathbb{R}^n$, a unit mass particle subject to the force given by a potential $V : \mathbb{R}^n \to \mathbb{R}$ moves according to Newton's law:

$$\gamma'' = -\nabla V(\gamma).$$

Now suppose that we have a hypersurface $M \subset \mathbb{R}^{n+1}$ and a smooth potential function $V : M \to \mathbb{R}$. We want to study the motion of a unit mass particle on $M$ subject to the resulting force. (i) What is the law of motion for $\gamma(t) \in M$? (ii) Now suppose that $f$ is a partial parametrization of $M$, with $V^f(x) = V(f(x))$, and write $\gamma(t) = f(c(t))$. What is equation for $c(t)$? Check that that equation is indeed invariant under reparametrizations (changing from one $f$ to another).