

## HOMEWORK 8

DUE: MONDAY, APRIL 10

This assignment contains only one homework problem to reinforce the notion of a spectral sequence. It will require Wednesday's lecture.

1. Let  $X$  be a CW complex, with skeletal filtration  $X^k$ . The skeletal filtration induces a filtration on the singular chain complex of  $X$ :

$$F_s C_*^{sing}(X) = C_*^{sing}(X^s).$$

Let  $\{E_{s,t}^r, d_r\}$  be the spectral sequence associated to this filtered complex. Identify the  $E^1$  term as a relative homology group. Argue that  $(E^1, d_1)$  is the cellular chain complex of  $X$ , and conclude that the spectral sequence collapses to give  $E^2 = E^\infty$ . Conclude that this recovers the isomorphism

$$H_*^{CW}(X) \cong H_*^{sing}(X).$$