18.727 Topics in Algebraic Geometry: Algebraic Surfaces
Spring 2008

Homework 1, 18.727 Spring 2008

1. Do the blowups necessary to reslove the the du Val singularities:
   (a) $A_4 : x^2 + y^2 + z^5 = 0$,
   (b) $D_5 : x^2 + y^2 z + z^4 = 0$,
   (c) $E_6 : x^2 + y^3 + z^4 = 0$,
   (d) $E_7 : x^2 + y^3 + yz^3 = 0$,
   (e) $E_8 : x^2 + y^3 + z^5 = 0$.
2. Show that every locally free sheaf of rank n on $\mathbb{P}^1$ is a isomorphic to a direct sum of n line bundles. (Hint: choose an invertible subsheaf of maximal degree.)
3. Prove the following proposition: Let $\pi : X \to B$ be a morphism from a (nonsingular projective) surface to a (nonsingular projective) curve, and let $D = \sum n_i E_i$ be a fiber of π. Then for every divisor $D' = \sum n_i' E_i$ with $n_i' \in \mathbb{Z}$ (i.e. supported on the fiber), we have $(D'^2) \leq 0$. If the fiber D is connected then $(D'^2) = 0$ if and only if $\exists a \in \mathbb{Q}$ such that $D' = aD$.