18.703 HOMEWORK #5, DUE THURSDAY MARCH 21ST

1. Herstein, Chapter 2, §6, 1.
2. Herstein, Chapter 2, §6, 2.
3. Herstein, Chapter 2, §6, 3& 4: (i) If G is a group and $N \lhd G$, show that if $\overline{M}$ is a subgroup of G/N and

$$M = \{ a \in G \mid Na \in \overline{M} \},$$

then M is a subgroup of G and $N \subset M$.
(ii) If in addition $\overline{M}$ is normal in G/N then M is normal in G.
4. Herstein, Chapter 2, §6, 7.
5. Herstein, Chapter 2, §6, 8.
6. Herstein, Chapter 2, §6, 11.
7. Herstein, Chapter 2, §6, 13.
8. Herstein, Chapter 2, §7, 2.
9. Herstein, Chapter 2, §7, 4.
10. Herstein, Chapter 2, §7, 6: If G is a group and $N \lhd G$, show that if $a \in G$ has finite order d, then aN in G/N has finite order m, where m divides d.
11. Herstein, Chapter 2, §7, 4.
12. Let H and K be two normal subgroups of a group G, whose intersection is the trivial subgroup. Prove that every element of H commutes with every element of K. (*Hint. Consider the commutator of an element of H and an element of K*).
13. Prove that a group G is isomorphic to the product of two groups H' and K' if and only if G contains two normal subgroups H and K, such that
   (i) H is isomorphic to H' and K is isomorphic to K'.
   (ii) $H \cap K = \{e\}$.
   (iii) $G = H \vee K$.
14. **Challenge Problem:** Find an example of a finite set, together with a binary operation, which satisfies all the axioms for a group, except associativity.

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