MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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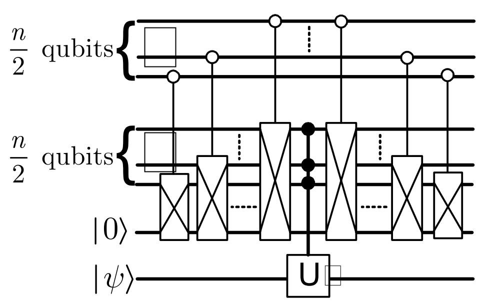
Quantum Computation

Problem 1. Find a circuit with $cn \log n$ gates that gives a good approximation to QFT on *n* qubits. (*c* is a constant.)

Problem 2. Problem 5.6 in Nielsen and Chuang. Show how to do addition using Fourier transform and phase shift.

Problem 3. In the Grover's algorithm, what is the probability of success after only one iteration if we are using two qubits (there are 4 possibilities) and there is only one right answer to the search problem. For the two-qubit system, the Grover's algorithm starts with $|\psi\rangle = |+\rangle \otimes |+\rangle$, and, in each iteration, we perform $(2|\psi\rangle\langle\psi| - I)O$, where O is the oracle operator that takes the right answer $|y\rangle$ to $-|y\rangle$ and leaves other states unchanged. The final measurement is in the computational basis.

Problem 4. For $n = 2^k$, we can use the following circuit, recursively, to build an *n*-qubit-controlled *U* gate using only single-qubit-controlled *U* gates and Fredkin gates with reverse polarity. Explain how this circuit works, and find how many gates and work bits will be needed to construct the controlled *U* gate.



where the Fredkin gate with reverse polarity swaps the two input states if the control qubit is $|0\rangle$ and does nothing if it is $|1\rangle$.