MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quantum Computation

Problem 1. For any unit vector $\mathbf{j} = (j_x, j_y, j_z)$ we can define the following operator

$$\sigma_{\mathbf{j}} = j_x\sigma_X + j_y\sigma_Y + j_z\sigma_Z$$

which corresponds to a π-radian rotation about **j**-axis.

(a) show that $\sigma_{\mathbf{j}}^2 = I$.
(b) $\sigma_{\mathbf{j}}$ has two eigenvalues: +1 and –1.
(c) Find the eigenvectors $|+\rangle_{\mathbf{j}}$ and $|-\rangle_{\mathbf{j}}$ respectively corresponding to eigenvalues +1 and –1.

Problem 2. Find an approximation to

$$e^{i\theta\sigma_Y/2}e^{i\theta\sigma_X/2}e^{-i\theta\sigma_Y/2}e^{-i\theta\sigma_X/2}$$

up to the second order of θ for $\theta \ll 1$.

Problem 3. Rewrite $\frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$ in the following basis

$$\{|+\rangle_A \otimes |+\rangle_B, |+\rangle_A \otimes |-\rangle_B, |-\rangle_A \otimes |+\rangle_B, |-\rangle_A \otimes |-\rangle_B\}.$$

What are $\Pr(++)$, $\Pr(+-)$, $\Pr(-+)$, and $\Pr(--)$?

Problem 4. For any two qubits $|\psi\rangle$ and $|\phi\rangle$, find a unitary operator U with the following property:

$$U|\psi\rangle_A \otimes |\phi\rangle_B = |\phi\rangle_A \otimes |\psi\rangle_B.$$

Write down U in the basis $\{|0\rangle_A \otimes |0\rangle_B, |0\rangle_A \otimes |1\rangle_B, |1\rangle_A \otimes |0\rangle_B, |1\rangle_A \otimes |1\rangle_B\}$.

Problem 5. Write out

$$\frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_C + |1\rangle_A \otimes |0\rangle_C) \otimes |+\rangle_B$$

in the following basis:

$$\{|000\rangle_{ABC}, |001\rangle_{ABC}, |010\rangle_{ABC}, |011\rangle_{ABC}, |100\rangle_{ABC}, |101\rangle_{ABC}, |110\rangle_{ABC}, |111\rangle_{ABC}\}.$$

Problem 6. Simplify the following expression:

$${}_A\langle +|(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)/\sqrt{2}.$$