MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.111J/18.435J/ESD.79
Quantum Computation

Problem 1. Prove that the Fredkin gate is universal. A Fredkin gate accepts three input bits A, B, and C, and gives three output bits D, E, and F in the following way:

D=A, E=B, and F=C if A=0.
D=A, E=C, and F=B if A=1.

A set of gates is called universal if you can build any logic circuits using theses gates assuming bit setting gates are given (that is, you can have as many as you wish work bits 0/1). Equivalently, you need to show that you can build NOT, AND, and Fan-out gates using the gates in the universal set.

Problem 2. Construct a binary adder using Toffoli gates (see page 29, Figure 1.14 for the Toffoli gate). This adder accepts two bits X and Y, and at the output, there is a carry bit as well as (X+Y mod 2).

Extra Credit: Construct such an adder assuming there is also a carry-in bit Z at the input.

Problem 3.

- **(a)** Verify that σ_i is equivalent to a rotation by π radians around *i*-axis in an RHS coordinate system for $i \in \{X, Y, Z\}$. (You basically need to find how σ_i operates on vectors $|0\rangle$, $|1\rangle$, $|+\rangle$, $|-\rangle$, $|\otimes\rangle$, and $|\bullet\rangle$.)
- **(b)** Show that $\sigma_X^2 = \sigma_Y^2 = \sigma_Z^2 = I$.
- **(c)** Show that $\sigma_X \sigma_Y = i\sigma_Z$, $\sigma_Y \sigma_Z = i\sigma_X$, and $\sigma_Z \sigma_X = i\sigma_Y$.
- **(d)** Verify that $\sigma_i \sigma_j + \sigma_j \sigma_i = 0$ for $i, j \in \{X, Y, Z\}$ and $i \neq j$.