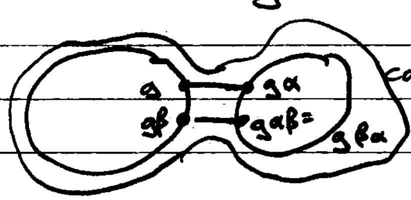
Pak

10/31/05

Thm G finite group, $G = \langle \alpha, \beta, \gamma \rangle$, $\alpha^2 = \beta^2 = \gamma^2 = 1$
$\Gamma = \Gamma(G, \{\alpha, \beta, \gamma\})$ Cayley graph $\alpha\beta = \beta\alpha$
contains h.c.

Def'n of Cayley graph
G finite gp
$S \subseteq G$
$\Gamma(G, S)$ is Cayley graph when
$V(\Gamma) = \{g \in G\}$ $E(\Gamma) = \{(g, gs) \mid s \in S\}$
(Clearly $\langle S \rangle = G \Leftrightarrow \Gamma$ is connected (yup, it's clear))

Pf of Thm: $H = \langle \beta, \gamma \rangle$ has Cayley graph big cycle w/ alternating β γ edges
Look at cosets of $\Gamma(H)$ w/in $\Gamma(G)$.
~~Note~~ Note they're 2-regular, each vx in $\Gamma(G)$ is 3-regular



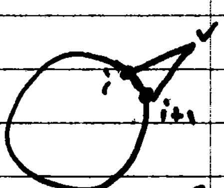
can hook up cycles like this, if not h.c. $\Rightarrow$
$\exists$ α edge going out $\Rightarrow$
hook up again $\Rightarrow$ ✓

Thm (Menger) (in MGT p75)
1) $s \neq t \in V(G) \Rightarrow$ min # vertices separating s & t (not = s or t)
= max # independent s-t paths (vertex disjoint except s & t)
2) min # edges separating s from t = max # edge disjoint s-t paths
3) $A, B \subseteq V$ min # ~~edges~~ vertices separating A from B
= max # vertex-disjoint paths from A to B

$3 \Rightarrow 1$ (Kind of) if $A = \{s\}$ $B = \{t\}$ then
min $= 1$ (s or t)
Better: let $A = N(s)$ and $B = N(t)$

$3 \Rightarrow 2$ Take $L(G)$ $V(L(G)) = E(G)$
$E(L(G)) = \{(e_1, e_2) \mid e_1 \cap e_2 \neq \emptyset\}$

Thm $|G| \geq 3$ $\kappa(G) \geq \alpha(G)$ ($\kappa(G)$ = vx-connectivity)
$\Rightarrow$ G contains h.c.
Pf Let C be the longest cycle, $v \in V - C$,
$P = \{P_1, P_2, \ldots\}$ max'l set of vertex-disjoint v-C paths
$|P| = \ell > \min\{|C|, \kappa(G)\}$ by M's Thm
Also $\ell < |C|/2$ $\Rightarrow$ $\ell \geq \kappa$. Can't have



or

since could then have longer cycles,
$\exists$ at least κ pts next to pts
connected to v $\Rightarrow$ they $+ v$ form
indep set of size $\kappa + 1$ ↯
(This is thm 10.1.2 in Diester's $G\theta$
(link on webpage))