HOMEWORK 4 (18.315, FALL 2005)

1) Let $G = K_{n_1..n_r}$ be r-partite graph with parts $n_1, \ldots, n_r$. Use the matrix-tree theorem to compute the number of spanning trees in G.

2) Problem 2.11 in Stanley, *EC1*. Use part b) to rederive the result of problem 1).

3) Problems 45, 46 from Bollobas, *MGT*, p. 376.

4) Let $G_{k,n}$ be the grid graph as before, and let $c(k,n,q)$ be the number of proper c-colorings of $G_{k,n}$ with q colors.

a) If k, n are fixed, prove that $c(k,n,q)$ can be computed in time polynomial in $(\log q)$.

b) If k, q are fixed, prove that $c(k,n,q)$ can be computed in time polynomial in n.

5) Let e_n be the expected number of cycles in a random permutation $\sigma\ S_n$. Compute e_n exactly. Conclude from here that $e_n = \theta(\log n)$.

6) Let $A(m)$ be the largest number of spanning trees a graph with m edges can have. Find non-trivial bounds on $A(m)$. (*Hint:* $A(m) \leq 2^m$ is a trivial bound. Similarly, a complete graph K_n gives a lower bound $A(m) \geq n^{n-2}$, where $m = \binom{n}{2}$. Can you do better?)

⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆