18.307 : Integral Equations :

NAME: Quiz 1 - Fall 2003

Pick-up Time:

Return Time:

18.307 Take–Home Quiz # 1

Thursday, October 16, 2003

You are given 5 hours to do the test.

You are NOT allowed to communicate with anyone, other than the instructor, about this quiz before 12 midnight today. Work on the test by yourself!

You can use any books or notes, but you are NOT allowed to use calculators, or any software such as Mathematica or Maple for example, in order to solve the problems.

Read the problems CAREFULLY. Justify your answers. Cross out what is not meant to be part of your final answer. Total # of points: 120.

I. (25 pts) Find the λ for which the integral equation (IE)

$$v(x) = \lambda \int_0^\infty dy\, e^{-2y}\,(y + e^x)\, v(y)$$

has non-trivial solutions. Give the corresponding $v(x)$.

II. (35 pts) The concentration $c = c(x,t)$ on the surface of a material obeys the partial differential equation (PDE)

$$\frac{\partial c}{\partial t} + \alpha \frac{\partial^4 c}{\partial x^4} = F(c), \quad t > 0, \quad -\infty < x < \infty, \quad \alpha > 0 : \text{const.},$$

and the <u>initial</u> condition $c(x,0) = g(x)$, where $g(x)$ is given, while $c(x,t)$ and its derivatives approach 0 as $|x| \to \infty$; $F(c)$ is a given bounded, nonlinear function of $c = c(x,t)$ which arises from the concentration-dependent deposition of material on the surface.

<u>Find</u> a suitable (causal) Green's function $G(x,t;x',t')$ for this problem, along with the corresponding solution c_0 of the homogeneous PDE; simplify the final expression for G as much as possible. <u>Convert</u> the given PDE to an IE; give this IE.

Hint: If you encounter any complicated integral(s) for G, try to express your results in terms of single (one-dimensional) integral(s), or in simpler form if possible.

III. (35 pts) Consider the integral equation (IE)

$$u(x) = x + \frac{1}{2}\int_0^A dy\,|y-x|\,u(y), \quad 0 < x < A.$$

(a)(20 pts). Solve the IE for $A = 1$.

(b)(15 pts) Does the given IE have a solution for $A = \infty$? Explain.

Hint: Beware: Since the lower limit of integration is 0 (or any finite number for that purpose), the application of Fourier transform is questionable. If you decide to "mutilate" the IE in any way, don't forget to get back to it when needed. In (b) you are NOT asked to solve the IE.

IV.(25 pts) (a)(18 pts) Solve entirely the following system of integral equations:

$$u_1(x) = u_2(x) + \lambda \int_0^x dy\, u_1(x-y)\, u_2(y) \qquad (1)$$

$$u_2(x) = 1 + \mu \int_0^x dy\, u_2(x-y)\, u_1(y), \qquad (2)$$

where $0 < x < \infty, \lambda$ and μ are given constants, and $u_1(x)$ and $u_2(x)$ are both unknown functions to be found, without using any iteration or recursion scheme.

(b)(7 pts) Discuss at least two distinct ways by which one can actually evaluate $u_1(x)$ and $u_2(x)$ as power series in x. One of these ways must follow from your solution of part (a).

Remark: In part (b) you are NOT asked to give the power series for u_1 and u_2 but just discuss clearly how one can get them from the solution found in (a), or by other means. In (b) you are allowed to use any alternative scheme you deem suitable.