18.175 PROBLEM SET EIGHT

A. Read and understand Chapters 7 and 8 of Durrett (or another text covering the same material). Write a few sentences of notes about your reading. (Hand them in, but they won't be graded. This is just to give you an excuse to take some notes.)

B. COMPLETE THE FOLLOWING PROBLEMS FROM DURRETT:

$$7.3.1, 7.3.2, 7.5.1, 7.5.2, 7.5.4, 8.1.2, 8.2.3, 8.5.2, 8.7.1$$

C. COMPLETE THE FOLLOWING PROBLEMS:

1. One way to construct an infinitely divisible random variable X supported on the rational numbers is as follows:

(a) Let N be a Poisson random variable with some parameter $\lambda > 0$.

(b) Let R be any random variable supported on the rationals. Let $R_1, R_2, \ldots$ be i.i.d. instances of R, independent of N.

(c) Let a be a fixed rational number.

(d) Write $X = a + R_1 + R_2 + \ldots + R_N$.

Prove that the X thus defined is infinitely divisible and that X is a.s. rational. Then answer the following: Can *every* infinitely divisible random variable that is a.s. rational be written in this way?

2. Let $\{X_i\}$ be a sequence of i.i.d. bounded random variables taking values in the integer grid $\mathbb{Z}^2$. Let $S_n = \sum_{i=1}^n X_i$. Prove that the sequence S_n is a recurrent Markov chain if and only if $\mathbb{E}[X_1] = 0$. Is this still true if we allow the X_i to be unbounded?

3. Can you give an example of a sequence of probability measures μ_n on $\mathbb{R}$ whose characteristic functions ϕ_n converge point-wise (as n tends to infinity) to the function $1_{\mathbb{Z}}$, where $\mathbb{Z}$ is the set of integers? What if we replace $1_{\mathbb{Z}}$ with 1_A, where A is the set of integers whose absolute values are perfect squares?

18.175 Theory of Probability
Spring 2014