

## Real analysis, Problem set 6, final project in harmonic analysis

Instead of a usual problem set, we finish the course with a more open-ended problem / project that touches on many of the topics from harmonic analysis that we have covered.

Consider the following operator defined for functions  $f : \mathbb{R} \rightarrow \mathbb{C}$ . We define  $Tf = f * K$ , where

$$K(x) = e^{ix^2} (1 + |x|^2)^{-1/4}.$$

We would like to find all the pairs  $(p, q)$  so that  $\|Tf\|_{L^q(\mathbb{R})} \lesssim \|f\|_{L^p(\mathbb{R})}$ . Figure out as much as you can about this problem. Can you find some  $(p, q)$  so that the estimate holds? Look at some examples! Can you find some  $(p, q)$  so that the estimate fails? Can you make a conjecture about exactly when the estimate holds? Many of the topics from our Fourier analysis unit could be helpful here: for example, interpolation, stationary phase, looking at different height scales or different frequency scales, ideas connected to Strichartz.

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