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18.112 Functions of a Complex Variable
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Problems for 18.112 Mid 2 (Open Book)

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1. (15') Evaluate

$$\int_{\gamma} \frac{dz}{e^z - 1}$$

where γ is the circle $|z| = 9$.

2. (30') Let $f(z)$ be analytic in the whole plane and assume that $\frac{\operatorname{Re}f(z)}{z} \rightarrow 0$ as $z \rightarrow \infty$. Prove: f is a constant.

(Hint: Use formula (66) valid for $|z| < R$:

$$f(z) = \frac{1}{2\pi i} \int_{|\zeta|=R} \frac{\zeta + z}{\zeta - z} u(\zeta) \frac{d\zeta}{\zeta} + iC, \quad u = \operatorname{Re}(x)$$

and estimate $f'(z)$ carefully for $z < \frac{R}{2}$ (Liouville).)

3. (25') If $f(z)$ is analytic for $|z| < 1$ and

$$|f(z)| \leq \frac{1}{1 - |z|},$$

find the best estimate of $f^{(n)}(0)$ that Cauchy's formula will yield.

(Hint: Use Cauchy's formula in each $|z| \leq r$, ($r < 1$).)

4. (30') How many roots does the equation

$$z^7 - 2z^5 + 6z^3 - z + 1 = 0$$

have in the disk $|z| < 1$? How many roots are inside $|z| = 2$?

(Hint: Look for the biggest term when $|z| = 1$, use Rouché.)