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18.112 Functions of a Complex Variable  
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# Problems for 18.112 Mid 1 (Open Book)

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1. (10') Find all solutions  $z$  to equation  $z^3 = -8i$ .

2. (15') Evaluate the integral

$$\int_{|z-1|=\frac{1}{2}} \frac{dz}{(1-z)^3}.$$

3. (20') Evaluate the integral

$$\int_{\gamma} \frac{e^z + z}{z-2} dz$$

in the two cases: 1)  $\gamma = \{z : |z| = 1\}$ ;    2)  $\gamma = \{z : |z| = 3\}$ .

4. (30') Let  $f(z)$  be analytic in the whole plane and assume it has a nonessential singularity at  $\infty$ . Show that  $f(z)$  is a polynomial.

5. (25') Suppose  $f(z)$  is analytic in the whole plane and suppose  $|f(z)| < |z|^n$  for some  $n$  and  $|z| > 100$ . Show that  $f$  is a polynomial.

(Hint: Look at  $f^{(m)}(0)$  for large  $m$ .)