

# Problem set 3

Turn in Monday 2008-01-28 in class. Turn in only the 'Problems' section. The other sections are for your own practice.

## Warmups

Warmup problems are quick problems for you to check your understanding; **don't turn them in**.

1. Draw a picture to show that

$$(x + y)^2 = x^2 + 2xy + y^2.$$

2. Estimate  $\sqrt{26}$  by taking out the big part.

## Problems

Turn in solutions to these problems.

3. Estimate  $\sqrt[3]{9}$ .
4. Use the small-angle approximation for  $\sin \theta$  to show that

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

for small  $\theta$ .

5. Riemann's zeta function

$$\zeta(s) = \sum_1^{\infty} \frac{1}{n^s}$$

is important for statistical physics, for the approximate analysis of random walks, for the theory of prime numbers, and for much else. In this problem you estimate  $\zeta(3/2)$ , which is the sum  $S = \sum_1^{\infty} n^{-3/2}$ .

- a. Sketch  $f(n) = n^{-3/2}$  and, on the same diagram, draw rectangles to illustrate the sum  $S$ .
  - b. Use the pictorial method to estimate the sum, and compare the estimate against the true value (approximately 2.612).
6. You want to cut a  $3 \times 3 \times 3$  cube into 27 unit cubes. What is the minimum number of knife cuts that you must make? No funky knife tricks: only planar cuts!

## Bonus problems

Bonus problems are more difficult but **optional** problems for those who are curious.

7. You want to cut a unit cube into two pieces each with volume  $1/2$ . What dividing surface, which might be curved, has the smallest surface area?