18.085 Computational Science and Engineering I
Fall 2008

18.085 Quiz 3 December 7, 2007 Professor Strang

Your PRINTED name is: _________________________ **Grading** **1**
2
3

** <u>**NOTE**</u> **AT NOON A BIG CHEMISTRY CLASS IS COMING !!!**

1) **(30 pts.)** (a) Solve by a Fourier sine series $u(x) = \sum b_k \sin kx$:

$$-u'' + 4\,u(x) = f(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & -\pi < x < 0 \end{cases} \quad \text{with} \quad u(-\pi) = u(\pi) = 0\,.$$

That right side $f(x)$ is the square wave SW(x) on page 318.

(b) What is the decay rate of the coefficients b_k? What is the smoothness of $u(x)$ — which derivative jumps?

2) **(30 pts.)** This problem is about the equation

$$\frac{1}{5}\,x_{n-1} + \frac{3}{5}\,x_n + \frac{1}{5}\,x_{n+1} = y_n \qquad -\infty < n < \infty$$

(a) Suppose the vector $x = (\ldots, x_{-1}, x_0, x_1, \ldots)$ is known. The equation is a non-cyclic convolution $a * x = y$. What is the infinite vector a? Transform the equation into the frequency domain using $X(\omega) = \sum x_n\, e^{in\omega}$ and $Y(\omega)$ and $A(\omega)$. What is $A(\omega)$ in this problem?

(b) Suppose the vector y is known but the vector x is **not known**. We want to find x. Take two steps:

1. Give a formula for $X(\omega)$ using known things like $Y(\omega)$ and $\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$, or A.
2. Does your formula involve any division by zero or is it safe?

The last step in this deconvolution would recover the Fourier coefficients x_n from your function $X(\omega)$ but this is not on the exam!

3) **(40 pts.)** This circulant equation $Cd = b$ is a cyclic convolution:

$$Cd = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = b \qquad \text{is} \quad c \circledast d = b$$

(a) The eigenvectors of that matrix C are the four columns e_0, e_1, e_2, e_3 of the Fourier matrix F (this F is on page 347). Multiply F times the e's to find the four eigenvalues. Check that their sum is correct.

(b) Write the right side $b = (1, 0, 0, 0)$ as a combination of those four eigenvectors (columns of F). Using the eigenvalues, the solution d is what combination of the four eigenvectors? Find the vector d.

(c) A direct way to solve $c \circledast d = b$ would be to take the 4-point discrete transform of both sides. What are the transforms of b and c in this problem? What is the transform of the solution d? Isn't this just the same method in different words (yes or no).

Thank you for taking 18.085!