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18.085 Computational Science and Engineering I
Fall 2008

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Your PRINTED name is: _____ Grading 1
2
3

- 1) (30 pts.) (a) Solve this *cyclic convolution* equation for the vector d . (I would transform convolution to multiplication.) Notice that $c = (5, 0, 0, 0) - (1, 1, 1, 1)$. The equation is like deconvolution.

$$c \circledast d = (4, -1, -1, -1) \circledast (d_0, d_1, d_2, d_3) = (1, 0, 0, 0).$$

- (b) Why is there no solution d if I change c to $C = (3, -1, -1, -1)$? Try it. Can you find a nonzero D so that $C \circledast D = (0, 0, 0, 0)$?

- 2) (36 pts.)
- (a) If $f(x) = e^{-x}$ for $0 \leq x \leq 2\pi$, extended periodically, find its (complex) Fourier coefficients c_k .
 - (b) What is the decay rate of those c_k and how could you see the decay rate from the function $f(x)$?
 - (c) Compute $\sum_{-\infty}^{\infty} |c_k|^2$ for those c 's as an ordinary number. [1 point question: How in the world could you find $\sum_{-\infty}^{\infty} |c_k|^4$? Don't try!]
 - (d) Solve this periodic differential equation to find $u(x)$:

$$u'(x) + u(x) = \delta(x) + \delta(x + 2\pi) + \delta(x - 2\pi) + \cdots \text{train of deltas}$$

3) (34 pts.) Suppose $f(x)$ is a *half-hat function* ($-\infty < x < \infty$).

$$f(x) = \begin{cases} 1 - x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for all other } x \end{cases}$$

- (a) Draw a graph of $f(x)$ on the whole line $-\infty < x < \infty$ and ALSO a graph of its derivative $g(x) = df/dx$.
- (b) What is the transform (Fourier integral) $\widehat{g}(k)$ of df/dx ?
- (c) What is the transform $\widehat{f}(k)$ of $f(x)$? Does it have the decay rate you expect? What is $\widehat{f}(0)$?
- (d) Christmas present: Is the convolution $f(x) * f(x)$ of the half-hat with itself equal to the usual full hat $H(x)$? (*Yes or no answer*, 4 points).

THANK YOU FOR TAKING 18.085! 18.086 will be good small projects in scientific computing.

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