18.085 Computational Science and Engineering I
Fall 2008

18.085 Quiz 1 October 5, 2007 Professor Strang

Your PRINTED name is: ____________________

Grading	**1**
	2
	3

1) **(39 pts.)** With $h = \frac{1}{3}$ there are 4 meshpoints $0, \frac{1}{3}, \frac{2}{3}, 1$ and displacements u_0, u_1, u_2, u_3.

a) Write down the matrices A_0, A_1, A_2 with three rows that produce the first differences $u_i - u_{i-1}$:

A_0 has 0 boundary conditions on u

A_1 has 1 boundary condition $u_0 = 0$ (left end fixed)

A_2 has 2 boundary conditions $u_0 = u_3 = 0$.

b) Write down all three matrices $A_0^{\mathrm{T}} A_0, A_1^{\mathrm{T}} A_1, A_2^{\mathrm{T}} A_2$.

CROSS OUT IF FALSE / GIVE REASON BASED ON COLUMNS OF A!

$K_0 = A_0^{\mathrm{T}} A_0$ is (singular) (invertible) (positive definite)

Reason:

$K_1 = A_1^{\mathrm{T}} A_1$ is (singular) (invertible) (positive definite)

Reason:

c) Find all solutions $w = (w_1, w_2, w_3)$ to each of these equations:

$$A_0^{\mathrm{T}} w = 0 \qquad A_1^{\mathrm{T}} w = 0 \qquad A_2^{\mathrm{T}} w = 0$$

2) **(33 pts.)**

a) Find the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and unit eigenvectors y_1, y_2, y_3 of B. Hint: one eigenvector is $(1, 0, -1)/\sqrt{2}$.

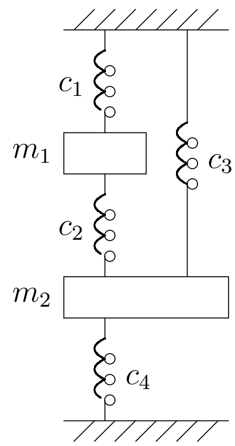
$$B = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

b) Factor B into $Q\Lambda Q^{\mathrm{T}}$ with $Q^{-1} = Q^{\mathrm{T}}$. Draw a graph of the energy function $f(u_1, u_2, u_3) = \frac{1}{2}u^{\mathrm{T}}Bu$. This is a surface in 4-dimensional u_1, u_2, u_3, f space so your graph may not be perfect—**OK to describe it in 1 sentence**.

c) What differential equation with what boundary conditions on $y(x)$ at $x = 0$ and 1 is the **continuous analog** of $By = \lambda y$? What are the eigenfunctions $y(x)$ and eigenvalues λ in this differential equation? At which x's would you sample the first three eigenfunctions to get the three eigenvectors in part (a)?

3) **(28 pts.)** The fixed-fixed figure shows $n = 2$ masses and $m = 4$ springs.

Displacements u_1, u_2.



a) Write down the stretching-displacement matrix A in $e = Au$.

b) What is the stiffness matrix $K = A^{\mathrm{T}}CA$ for this system ?

c) **Theory question about any $\boldsymbol{A^{\mathrm{T}}CA}$.** C is symmetric positive definite. What condition on A assures that $u^{\mathrm{T}}A^{\mathrm{T}}CAu > 0$ for every vector $u \neq 0$? Explain why this is greater than zero and where you use your condition on A.