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18.085 Computational Science and Engineering I  
Fall 2008

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Your PRINTED name is: \_\_\_\_\_

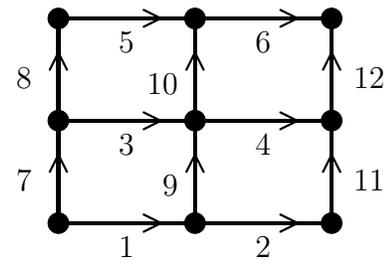
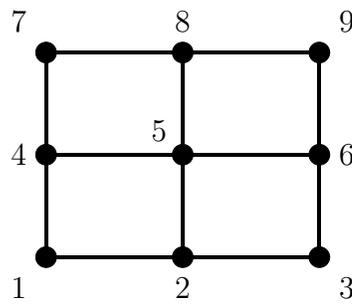
Grading 1

2

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1) (40 pts.) Here is a network with 9 numbered nodes and then separately the 12 edges.



- What is the shape of the incidence matrix  $A$ ? What is its 4th row?
- What is the 5th column of  $A$ ? What is the  $(5, 5)$  entry in  $A^T A$ ? Write the whole 5th row of  $A^T A$ .
- How many independent solutions to Kirchhoff's Law  $A^T w = 0$ ? Find one of them.
- Without writing down this matrix  $A$ , explain why  $A^T A$  is or is not symmetric positive definite.



- 2) (30 pts.)
- (a) For a hanging elastic bar, with  $u(0) = 0$  at the top and  $u'(1) = 0$  at the bottom and elastic constant  $c(x) = 1$ , what is the displacement  $u(x)$  when a *unit point load*  $f(x) = \delta(x - a)$  acts at the point  $x = a$ ? Draw a graph of  $u(x)$ .
  - (b) What is the limit of  $u(x)$  as the unit load moves to the bottom ( $a \rightarrow 1$ )? Suppose it moves to the top ( $a \rightarrow 0$ )? Draw graphs of  $u(x)$  in those two cases.
  - (c) Choose a matrix equation that approximates the differential equation in part (a). (Describe the matrix—OK to put the load at a meshpoint.) If the load moves to the lowest meshpoint (number  $N$ ), what displacements correspond to your answer in part (b)?



- 3) (30 pts.) Suppose you measure your initial position  $u_1 = b_1$ , and then you measure the step lengths  $u_2 - u_1 = b_2$  and  $u_3 - u_2 = b_3$ . At the end you make a last measurement  $u_3 = b_4$ .
- (a) Under what conditions on  $b_1, b_2, b_3, b_4$  will these four equations have an exact solution? Create (don't solve) a set of equations for the best estimates  $\hat{u}_1, \hat{u}_2, \hat{u}_3$ .
- (b) Draw a picture of masses, springs, and forces (write in all constants) that would lead to the same equations for the displacements.
- (c) Suppose the variances for errors in the measurements are  $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2$ . What equations should you solve (DON'T DO IT) for the statistically best estimate  $\hat{u}$ ? If  $\sigma_4 \rightarrow \infty$  so that  $b_4$  becomes completely unreliable, what answer do you expect for the best  $\hat{u}$ ?

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