18.085 Computational Science and Engineering I
Fall 2008

1 Question 1

If you ordered the bars in the same order as the nodes (starting with bar 1 between nodes 1 and 2),

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

You'd still get credit if you used a different order.

There were many ways to give an independent set of solutions. Here's one independent set:

- the entire truss moves to the right
- the entire truss moves up
- the truss rotates clockwise about the origin
- nodes 1 and 3 move out, nodes 2 and 4 move in

The vectors corresponding to those solutions would be the columns of this matrix:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

2 Question 2

Multiplying the equation by $v(x)$ and integrating from 0 to 1,

$$\left[-e^x \frac{du}{dx} v(x)\right]_0^1 + \int_0^1 e^x \frac{du}{dx}\frac{dv}{dx} dx = v(a)$$

If you drop the first term, you get the weak form. This first term is already zero at $x = 0$ because of the boundary conditions on $u(x)$. To make it be zero at $x = 1$, you need to impose

$$v(1) = 0$$

which both test functions in this example satisfy.

Substituting $u(x) = U_1\phi_1(x) + U_2\phi_2(x)$ and both $v(x) = V_1(x)$ and $v(x) = V_2(x)$ into the weak form above gives a system of equations $KU = F$ where

$$K = \begin{pmatrix} \frac{e^a-1}{a^2} & \frac{1-e^a}{a^2} \\ \frac{1-e^a}{a^2} & \frac{e^a-1}{a^2} + \frac{e-e^a}{(1-a)^2} \end{pmatrix}$$

$$F = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

K_e is the same as K above except with the integrals performed only over the interval 0 to a. The result is the same except the lower right entry is missing the second term.