

### Exercises on Cramer's rule, inverse matrix, and volume

**Problem 20.1:** (5.3 #8. *Introduction to Linear Algebra*: Strang) Suppose

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}.$$

Find its cofactor matrix  $C$  and multiply  $AC^T$  to find  $\det(A)$ .

$$C = \begin{bmatrix} 6 & -3 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \text{ and } AC^T = \text{_____}.$$

If you change  $a_{1,3} = 4$  to 100, why is  $\det(A)$  unchanged?

**Problem 20.2:** (5.3 #28.) Spherical coordinates  $\rho, \phi, \theta$  satisfy

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta \text{ and } z = \rho \cos \phi.$$

Find the three by three matrix of partial derivatives:

$$\begin{bmatrix} \partial x / \partial \rho & \partial x / \partial \phi & \partial x / \partial \theta \\ \partial y / \partial \rho & \partial y / \partial \phi & \partial y / \partial \theta \\ \partial z / \partial \rho & \partial z / \partial \phi & \partial z / \partial \theta \end{bmatrix}.$$

Simplify its determinant to  $J = \rho^2 \sin \phi$ . In spherical coordinates,

$$dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

is the volume of an infinitesimal "coordinate box."

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