

Exercises on the four fundamental subspaces

Problem 10.1: (3.6 #11. *Introduction to Linear Algebra: Strang*) A is an m by n matrix of rank r . Suppose there are right sides \mathbf{b} for which $A\mathbf{x} = \mathbf{b}$ has *no solution*.

- a) What are all the inequalities ($<$ or \leq) that must be true between m , n , and r ?
- b) How do you know that $A^T\mathbf{y} = \mathbf{0}$ has solutions other than $\mathbf{y} = \mathbf{0}$?

Solution:

- a) The rank of a matrix is always less than or equal to the number of rows and columns, so $r \leq m$ and $r \leq n$. The second statement tells us that the column space is not all of \mathbb{R}^n , so $r < n$.
- b) These solutions make up the left nullspace, which has dimension $m - r > 0$ (that is, there are nonzero vectors in it).

Problem 10.2: (3.6 #24.) $A^T\mathbf{y} = \mathbf{d}$ is solvable when \mathbf{d} is in which of the four subspaces? The solution \mathbf{y} is unique when the _____ contains only the zero vector.

Solution: It is solvable when \mathbf{d} is in the row space, which consists of all vectors $A^T\mathbf{y}$. The solution \mathbf{y} is unique when the **left nullspace** contains only the zero vector.

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18.06SC Linear Algebra
Fall 2011

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