

18.06SC Unit 1 Exam

1 (24 pts.) This question is about an m by n matrix A for which

$$Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ has no solutions} \quad \text{and} \quad Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ has exactly one solution.}$$

- (a) Give all possible information about m and n and the rank r of A .
- (b) Find all solutions to $Ax = 0$ and **explain your answer**.
- (c) Write down an example of a matrix A that fits the description in part (a).

2 (24 pts.) The 3 by 3 matrix A reduces to the identity matrix I by the following three row operations (in order):

E_{21} : Subtract 4 (row 1) from row 2.

E_{31} : Subtract 3 (row 1) from row 3.

E_{23} : Subtract row 3 from row 2.

- (a) Write the inverse matrix A^{-1} in terms of the E 's. **Then compute A^{-1} .**
- (b) What is the original matrix A ?
- (c) What is the lower triangular factor L in $A = LU$?

3 (28 pts.) This 3 by 4 matrix depends on c :

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 3 & c & 2 & 8 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

(a) *For each c* find a basis for the column space of A .

(b) *For each c* find a basis for the nullspace of A .

(c) *For each c* find the complete solution x to $Ax = \begin{bmatrix} 1 \\ c \\ 0 \end{bmatrix}$.

- 4 (24 pts.) (a) If A is a 3 by 5 matrix, what information do you have about the nullspace of A ?
- (b) Suppose row operations on A lead to this matrix $R = \text{rref}(A)$:

$$R = \begin{bmatrix} 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Write all known information about the columns of A .

- (c) In the vector space M of all 3 by 3 matrices (you could call this a matrix space), what subspace S is spanned by all possible row reduced echelon forms R ?

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