

Exercises on column space and nullspace

Problem 6.1: (3.1 #30. *Introduction to Linear Algebra: Strang*) Suppose \mathbf{S} and \mathbf{T} are two subspaces of a vector space \mathbf{V} .

- a) **Definition:** The sum $\mathbf{S} + \mathbf{T}$ contains all sums $\mathbf{s} + \mathbf{t}$ of a vector \mathbf{s} in \mathbf{S} and a vector \mathbf{t} in \mathbf{T} . Show that $\mathbf{S} + \mathbf{T}$ satisfies the requirements (addition and scalar multiplication) for a vector space.
- b) If \mathbf{S} and \mathbf{T} are lines in \mathbf{R}^m , what is the difference between $\mathbf{S} + \mathbf{T}$ and $\mathbf{S} \cup \mathbf{T}$? That union contains all vectors from \mathbf{S} and \mathbf{T} or both. Explain this statement: *The span of $\mathbf{S} \cup \mathbf{T}$ is $\mathbf{S} + \mathbf{T}$.*

Problem 6.2: (3.2 #18.) The plane $x - 3y - z = 12$ is parallel to the plane $x - 3y - x = 0$. One particular point on this plane is $(12, 0, 0)$. All points on the plane have the form (fill in the first components)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Problem 6.3: (3.2 #36.) How is the nullspace $\mathbf{N}(C)$ related to the spaces $\mathbf{N}(A)$ and $\mathbf{N}(B)$, if $C = \begin{bmatrix} A \\ B \end{bmatrix}$?

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18.06SC Linear Algebra
Fall 2011

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