

MATLAB Tutorial

You need a small number of basic commands to start using MATLAB. This short tutorial describes those fundamental commands. You need to *create* vectors and matrices, to *change* them, and to *operate* with them. Those are all short high-level commands, because MATLAB constantly works with matrices. I believe that you will like the power that this software gives, to do linear algebra by a series of short instructions:

| | | | |
|---|---|---|---|
| <i>create E</i> | <i>create u</i> | <i>change E</i> | <i>multiply Eu</i> |
| $E = \text{eye}(3)$ | $u = E(:, 1)$ | $E(3, 1) = 5$ | $v = E * u$ |
| $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ |

The word *eye* stands for the identity matrix. The submatrix $u = E(:, 1)$ picks out column 1. The instruction $E(3, 1) = 5$ resets the (3, 1) entry to 5. The command $E * u$ multiplies the matrices E and u . All these commands are repeated in our list below. Here is an example of inverting a matrix and solving a linear system:

| | | | |
|---|---|---|---|
| <i>create A</i> | <i>create b</i> | <i>invert A</i> | <i>solve Ax = b</i> |
| $A = \text{ones}(3) + \text{eye}(3)$ | $b = A(:, 3)$ | $C = \text{inv}(A)$ | $x = A \backslash b$ or $x = C * b$ |
| $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ | $\begin{bmatrix} .75 & -.25 & -.25 \\ -.25 & .75 & -.25 \\ -.25 & -.25 & .75 \end{bmatrix}$ | $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ |

The matrix of all ones was added to $\text{eye}(3)$, and b is its third column. Then $\text{inv}(A)$ produces the inverse matrix (normally in decimals; for fractions use *format rat*). The system $Ax = b$ is solved by $x = \text{inv}(A) * b$, which is the slow way. The backslash command $x = A \backslash b$ uses Gaussian elimination if A is square and never computes the inverse matrix. When the right side b equals the third column of A , the solution x must be $[0 \ 0 \ 1]'$. (*The transpose symbol ' makes x a column vector.*) Then $A * x$ picks out the third column of A , and we have $Ax = b$.

Here are a few comments. The comment symbol is `%`:

`%` The symbols a and A are *different*: MATLAB is case-sensitive.

`%` Type *help slash* for a description of how to use the backslash symbol. The word *help* can be followed by a MATLAB symbol or command name or M-file name.

Note: The command name is upper case in the description given by help, but must be lower case in actual use. And the backslash $A \setminus b$ is different when A is not square.

% To display all 16 digits type *format long*. The normal *format short* gives 4 digits after the decimal.

% A semicolon after a command avoids display of the result.

$A = \text{ones}(3)$; will not display the 3×3 identity matrix.

% Use the up-arrow cursor to return to previous commands.

How to input a row or column vector

$u = [2\ 4\ 5]$ has one row with three components (a 1×3 matrix)

$v = [2; 4; 5]$ has three rows separated by semicolons (a 3×1 matrix)

$v = [2\ 4\ 5]'$ or $v = u'$ *transposes* u to produce the same v

$w = 2:5$ generates the row vector $w = [2\ 3\ 4\ 5]$ with unit steps

$u = 1:2:7$ takes steps of 2 to give $u = [1\ 3\ 5\ 7]$

How to input a matrix (a row at a time)

$A = [1\ 2\ 3; 4\ 5\ 6]$ has two rows (always a semicolon between rows)

$A = [1\ 2\ 3$
4 5 6]

 also produces the matrix A but is harder to type

$B = [1\ 2\ 3; 4\ 5\ 6]'$ is the *transpose* of A . Thus A^T is A' in MATLAB

How to create special matrices

diag(v) produces the diagonal matrix with vector v on its diagonal

toeplitz(v) gives the symmetric *constant-diagonal* matrix with v as first row and first column

toeplitz(w, v) gives the constant-diagonal matrix with w as first column and v as first row

ones(n) gives an $n \times n$ matrix of ones

zeros(n) gives an $n \times n$ matrix of zeros

eye(n) gives the $n \times n$ identity matrix

rand(n) gives an $n \times n$ matrix with random entries between 0 and 1 (uniform distribution)

randn(n) gives an $n \times n$ matrix with normally distributed entries (mean 0 and variance 1)

ones(m, n) **zeros**(m, n) **rand**(m, n) give $m \times n$ matrices

ones(**size**(A)) **zeros**(**size**(A)) **eye**(**size**(A)) give matrices of the same shape as A

How to change entries in a given matrix A

$A(3, 2) = 7$ resets the $(3, 2)$ entry to equal 7

$A(3, :) = v$ resets the third row to equal v

$A(:, 2) = w$ resets the second column to equal w

The colon symbol $:$ stands for *all* (all columns or all rows)

$A([2\ 3], :) = A([3\ 2], :)$ exchanges rows 2 and 3 of A

How to create submatrices of an $m \times n$ matrix A

$A(i, j)$ returns the (i, j) entry of the matrix A (scalar = 1×1 matrix)

$A(i, :)$ returns the i th row of A (as row vector)

$A(:, j)$ returns the j th column of A (as column vector)

$A(2 : 4, 3 : 7)$ returns rows from 2 to 4 and columns from 3 to 7 (as 3×5 matrix)

$A([2\ 4], :)$ returns rows 2 and 4 and all columns (as $2 \times n$ matrix)

$A(:)$ returns one long column formed from the columns of A ($mn \times 1$ matrix)

triu(A) sets all entries below the main diagonal to zero (upper triangular)

tril(A) sets all entries above the main diagonal to zero (lower triangular)

Matrix multiplication and inversion

$A * B$ gives the matrix product AB (if A can multiply B)

$A .* B$ gives the entry-by-entry product (if $\text{size}(A) = \text{size}(B)$)

inv(A) gives A^{-1} if A is square and invertible

pinv(A) gives the pseudoinverse of A

$A \setminus B$ gives **inv**(A) * B if **inv**(A) exists: *backslash* is left division

$x = A \setminus b$ gives the solution to $Ax = b$ if **inv**(A) exists

See *help slash* when A is a rectangular matrix!

Numbers and matrices associated with A

$\det(A)$ is the *determinant* (if A is a square matrix)

$\text{rank}(A)$ is the *rank* (number of pivots = dimension of row space and of column space)

$\text{size}(A)$ is the pair of numbers $[m \ n]$

$\text{trace}(A)$ is the *trace* = sum of diagonal entries = sum of eigenvalues

$\text{null}(A)$ is a matrix whose $n - r$ columns are an orthogonal basis for the nullspace of A

$\text{orth}(A)$ is a matrix whose r columns are an orthogonal basis for the column space of A

Examples

$E = \text{eye}(4); E(2, 1) = -3$ creates a 4×4 elementary elimination matrix

$E * A$ subtracts 3 times row 1 of A from row 2.

$B = [A \ b]$ creates the augmented matrix with b as extra column

$E = \text{eye}(3); P = E([2 \ 1 \ 3], :)$ creates a permutation matrix

Note that $\text{triu}(A) + \text{tril}(A) - \text{diag}(\text{diag}(A))$ equals A

Built-in M-files for matrix factorizations (all important!)

$[L, U, P] = \text{lu}(A)$ gives three matrices with $PA = LU$

$e = \text{eig}(A)$ is a vector containing the eigenvalues of A

$[S, E] = \text{eig}(A)$ gives a diagonal eigenvalue matrix E and eigenvector matrix S with $AS = SE$. If A is not diagonalizable (too few eigenvectors) then S is not invertible.

$[Q, R] = \text{qr}(A)$ gives an $m \times m$ orthogonal matrix Q and $m \times n$ triangular R with $A = QR$

Creating M-files

M-files are text files ending with `.m` which MATLAB uses for functions and scripts. A script is a sequence of commands which may be executed often, and can be placed in an m-file so the commands do not have to be retyped. MATLAB's demos are examples of these scripts. An example is the demo called *house*. Most of MATLAB's functions are actually m-files, and can be viewed by writing `type xxx` where *xxx* is the name of the function.

To write your own scripts or functions, you have to create a new text file with any name you like, provided it ends with `.m`, so MATLAB will recognize it. Text files can be created, edited and saved with any text editor, like *emacs*, *EZ*, or *vi*. A script file is simply a list of MATLAB commands. When the file name is typed at the MATLAB prompt, the contents of the file will be executed. For an m-file to be a function it must start with the word *function* followed by the output variables in brackets, the function name, and the input variables.

Examples

```
function [C]=mult(A)
r=rank(A);
C = A' * A;
```

Save the above commands into a text file named `mult.m`. Then this function will take a matrix A and return only the matrix product C . The variable r is not returned because it was not included as an output variable. The commands are followed by `;` so that they will not be printed to the MATLAB window every time they are executed. It is useful when dealing with large matrices. Here is another example:

```
function [V,D,r]=properties(A)
% This function finds the rank, eigenvalues and eigenvectors of A
[m,n]=size(A);
if m==n
[V,D]=eig(A);
r=rank(A);
else
disp('Error: The matrix must be square');
end
```

Here the function takes the matrix A as input and only returns two matrices and the rank as output. The `%` is used as a comment. The function checks to see if the input matrix is square and then finds the rank, eigenvalues and eigenvectors of a matrix A . Typing `properties(A)` only returns the first output, V , the matrix of eigenvectors. You must type `[V,D,r]=properties(A)` to get all three outputs.

Keeping a diary of your work

The command **diary**('file') tells MATLAB to record everything done in the MATLAB window, and save the results in the text file named 'file'. Typing **diary on** or **diary off** toggles the recording. Old diary files can be viewed using a text editor, or printed using *lpr* in unix. In MATLAB, they can be viewed using the **type file** command.

Saving your variables and matrices

The command **diary** saves the commands you typed as well as MATLAB's output, but it does not save the content of your variables and matrices. These variables can be listed by the command **whos** which also lists the sizes of the matrices. The command **save 'xxx'** will save the matrices and all variables listed by the **whos** command into the file named *xxx*. MATLAB labels these files with a .mat extension instead of .m which are scripts or functions. *xxx.mat* files can be read by MATLAB at a later time by typing **load xxx**.

Graphics

The simplest command is **plot**(x, y) which uses two vectors x and y of the same length. The points (x_i, y_i) will be plotted and connected by solid lines.

If no vector x is given, MATLAB assumes that $x(i) = i$. Then **plot**(y) has equal spacing on the x -axis: the points are $(i, y(i))$.

The type and color of the line between points can be changed by a third argument. The default with no argument is a solid black line “-”. Use *help plot* for many options, we indicate only a few:

MATLAB 5: **plot**($x, y, 'r+ :'$) plots in $r =$ red with $+$ for points and dotted line

MATLAB 4: **plot**($x, y, '- -'$) is a dashed line and **plot**($x, y, ' .'$) is a dotted line

You can omit the lines and plot only the discrete points in different ways:

plot($x, y, 'o'$) gives circles. Other options are $'+'$ or $'\times'$ or $'*'$

For two graphs on the same axes use **plot**(x, y, X, Y). Replace **plot** by **loglog** or **semilogy** or **semilogx** to change one or both axes to logarithmic scale. The command

axis (*a b c d*) will scale the graph to lie in the rectangle $a \leq x \leq b, c \leq y \leq d$. To title the graph or label the *x*-axis or the *y*-axis, put the desired label in quotes as in these examples:

title ('height of satellite') *xlabel* ('time in seconds') *ylabel* ('height in meters')

The command **hold** keeps the current graph as you plot a new graph. Repeating **hold** will clear the screen. To print, or save the graphics window in a file, see *help print* or use

print -Pprintername print -d filename

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