

Exam 2 Practice Questions, 18.05, Spring 2014

Note: This is a set of practice problems for exam 2. The actual exam will be much shorter. Within each section we've arranged the problems roughly in order of difficulty.

1 Topics

- Statistics: data, MLE (pset 5)
- Bayesian inference: prior, likelihood, posterior, predictive probability, probability intervals (psets 5, 6)
- Frequentist inference: NHST (psets 7, 8)

2 Using the probability tables

You should become familiar with the probability tables at the end of these notes.

1. Use the standard normal table to find the following values. In all the problems Z is a standard normal random variable.

- (a) (i) $P(Z < 1.5)$ (ii) $P(Z > 1.5)$ (iii) $P(-1.5 < Z < 1.5)$
(iv) $P(Z \leq 1.625)$

- (b) (i) The right-tail with probability $\alpha = 0.05$.
(ii) The two-sided rejection region with probability $\alpha = 0.2$.
(iii) Find the range for the middle 50% of probability.

2. To save space the t -tables only give probabilities for $t > 0$. You need to use the symmetry of the t -distribution to get probabilities when $t < 0$.

Use the t -tables table to find the following values. In all the problems T is a random variable drawn from a t -distribution with the indicated number of degrees of freedom.

- (a) (i) $P(T < 1.6)$, with $df = 3$
(ii) $P(T > 1.6)$ with $df = 10$
(iii) $P(-1.6 < T < 1.6)$ with $df = 49$
(iv) $P(T \leq 1.7)$ with $df = 40$
- (b) (i) The critical value for probability $\alpha = 0.05$ for 8 degrees of freedom.
(ii) The two-sided rejection region with probability $\alpha = 0.2$ for 16 degrees of freedom.
(iii) Find the range for the middle 50% of probability with $df = 20$.

3. The chi-square tables are different. They give the right critical values corresponding to probabilities.

Use the chi-square tables table to find the following values. In all the problems X^2 is a random variable drawn from a χ^2 -distribution with the indicated number of degrees of freedom.

(a) (i) $P(X^2 > 1.6)$, with $df = 3$

(ii) $P(X^2 > 20)$ with $df = 16$

(b) (i) The right critical value for probability $\alpha = 0.05$ for 8 degrees of freedom.

(ii) The two-sided rejection region with probability $\alpha = 0.2$ for 16 degrees of freedom.

3 Data

4. The following data is from a random sample: 5, 1, 3, 3, 8.

Compute the sample mean, sample standard deviation and sample median.

4 MLE

5. (a) A coin is tossed 100 times and lands heads 62 times. What is the maximum likelihood estimate for θ the probability of heads.

(b) A coin is tossed n times and lands heads k times. What is the maximum likelihood estimate for θ the probability of heads.

6. Suppose the data set y_1, \dots, y_n is a drawn from a random sample consisting of i.i.d. discrete uniform distributions with range 1 to N . Find the maximum likelihood estimate of N .

7. Suppose data x_1, \dots, x_n is drawn from an exponential distribution $\exp(\lambda)$. Find the maximum likelihood for λ .

8. Suppose x_1, \dots, x_n is a data set drawn from a Geometric($1/a$) distribution. Find the maximum likelihood estimate of a .

9. You want to estimate the size of an MIT class that is closed to visitors. You know that the students are numbered from 1 to n , where n is the number of students. You call three random students out of the classroom and ask for their numbers, which turn out to be 1, 3, 7. Find the maximum likelihood estimate for n . (Hint: the student #'s are drawn from a discrete uniform distribution.)

5 Bayesian updating: discrete prior, discrete likelihood

10. Twins Suppose $1/3$ of twins are identical and $2/3$ of twins are fraternal. If you are pregnant with twins of the same sex, what is the probability that they are identical?

11. Dice. You have a drawer full of 4, 6, 8, 12 and 20-sided dice. You suspect that they are in proportion 1:2:10:2:1. Your friend picks one at random and rolls it twice getting 5 both times.

(a) What is the probability your friend picked the 8-sided die?

(b) (i) What is the probability the next roll will be a 5?

(ii) What is the probability the next roll will be a 15?

12. Sameer has two coins: one fair coin and one biased coin which lands heads with probability $3/4$. He picks one coin at random (50-50) and flips it repeatedly until he gets a tails. Given that he observes 3 heads before the first tails, find the posterior probability that he picked each coin.

(a) What are the prior and posterior odds for the fair coin?

(b) What are the prior and posterior predictive probabilities of heads on the next flip? Here prior predictive means prior to considering the data of the first four flips.

6 Bayesian Updating: continuous prior, discrete likelihood

13. Jon and Jerry disagree over whether 18.05 students prefer Bayesian or frequentist statistics. They decide to pick a random sample of 10 students from the class and get Shelby to ask each student which they prefer. They agree to start with a prior $f(\theta) \sim \text{beta}(2, 2)$, where θ is the percent that prefer Bayesian.

(a) Let x_1 be the number of people in the sample who prefer Bayesian statistics. What is the pmf of x_1 ?

(b) Compute the posterior distribution of θ given $x_1 = 6$.

(c) Use R to compute 50% and 90% probability intervals for θ . Center the intervals so that the leftover probability in both tails is the same.

(d) The *maximum a posteriori* (MAP) estimate of θ (the peak of the posterior) is given by $\hat{\theta} = 7/12$, leading Jerry to concede that a majority of students are Bayesians. In light of your answer to part (c) does Jerry have a strong case?

(e) They decide to get another sample of 10 students and asks Shelby to poll them. Write down in detail the expression for the posterior predictive probability that the

majority of the second sample prefer Bayesian statistics. The result will be an integral with several terms. Don't bother computing the integral.

7 Bayesian Updating: discrete prior, continuous likelihood

14. Suppose that Alice is always X hours late to class and X is uniformly distributed on $[0, \theta]$. Suppose that *a priori*, we know that θ is either $1/4$ or $3/4$, both equally likely. If Alice arrives 10 minutes late, what is the most likely value of θ ? What if she had arrived 30 minutes late?

8 Bayesian Updating: continuous prior, continuous likelihood

15. Suppose that you have a cable whose exact length is θ . You have a ruler with known error normally distributed with mean 0 and variance 10^{-4} . Using this ruler, you measure your cable, and the resulting measurement x is distributed as $N(\theta, 10^{-4})$.

(a) Suppose your prior on the length of the cable is $\theta \sim N(9, 1)$. If you then measure $x = 10$, what is your posterior pdf for θ ?

(b) With the same prior as in part (a), compute the total number of measurements needed so that the posterior variance of θ is less than 10^{-6} .

16. **Gamma prior.** Customer waiting times (in hours) at a popular restaurant can be modeled as an exponential random variable with parameter λ . Suppose that *a priori* we know that λ can take any value in $(0, \infty)$ and has density function

$$f(\lambda) = \frac{1}{4!} \lambda^4 e^{-\lambda}.$$

Suppose we observe 5 customers, with waitings times $x_1 = 0.23, x_2 = 0.80, x_3 = 0.12, x_4 = 0.35, x_5 = 0.5$. Compute the posterior density function of λ .

(Hint: $\int_0^\infty y^{a-1} e^{-by} dy = \frac{(a-1)!}{b^a}$.)

17. **Exponential censoring.** [*Information Theory, Inference, and Learning Algorithms* by David J. C. Mackay]. Unstable particles are emitted from a source and decay at a distance $X \sim \exp(\lambda)$, where λ is unknown. Scientists are interested in finding the mean decay distance, given by $1/\lambda$. Their equipment is such that decay events can be observed only if they occur in a window extending from $x = 1\text{cm}$ to $x = 20\text{cm}$.

(a) Let $Z(\lambda)$ be the probability that an emitted particle decays in the window of detection. Find $Z(\lambda)$ in terms of λ .

(b) A decay event is observed at location x . Find the likelihood $f(x|\lambda)$. Hint: This is the probability that an *observed* decay event occurs at location x , given λ . Use (a).

(c) Suppose that based on earlier experiments, scientists believe that the mean decay distance $1/\lambda$ is equally likely to be anywhere between 5cm and 30cm. By transforming random variables, this corresponds to a prior for λ of $f_{\Lambda}(\lambda) = \frac{1}{25\lambda^2}$ on $[\frac{1}{30}, \frac{1}{5}]$. Over the course of a new experiment, 4 decay events are observed at locations $\{5, 11, 13, 14\}$. Find the posterior odds that the mean decay distance is greater than 10cm (i.e., $\lambda \leq \frac{1}{10}$). Express your answer as a ratio of two integrals (you do not need to evaluate these integrals; in practice you would hand them to a computer). Using the R function `integrate()` we computed Odds ($\frac{1}{\lambda} > 10$) ≈ 10.1 .

9 NHST

18. z -test

Suppose we have 49 data points with sample mean 6.25 and sample variance 12. We want to test the following hypotheses

H_0 : the data is drawn from a $N(4, 10^2)$ distribution.

H_A : the data is drawn from $N(\mu, 10^2)$ where $\mu \neq 4$.

(a) Test for significance at the $\alpha = 0.05$ level. Use the tables at the end of this file to compute p -values.

(b) Draw a picture showing the null pdf, the rejection region and the area used to compute the p -value.

19. t -test, χ^2 -test

Suppose we have 49 data points with sample mean 6.25 and sample variance 36. We want to test the following hypotheses:

(a) H_0 : the data is drawn from $N(4, \sigma^2)$, where σ is unknown.

H_A : the data is drawn from $N(\mu, \sigma^2)$ where $\mu \neq 4$.

Test for significance at the $\alpha = 0.05$ level. Use the t -table to find the p value.

(b) Draw pictures showing the null pdf, the rejection region and the area used to compute the p -value for parts (a) and (b).

20. Probability, MLE, goodness of fit

An experiment tested the rate of success for a certain medical procedure. The researchers ran n trials and generated data x_1, \dots, x_n . Unfortunately the experimental protocol was lost. We know that x is the number of successes, but we don't know the details of the experiment. It could have been:

Experiment 1: The procedure was tried 12 times and x is the total number of successes.

Experiment 2: The procedure was tried until the first failure and x is the total number of successes.

Let p be the probability of success.

(a) (i) Give the pmf for one trial of Experiment 1.

(ii) Give the pmf for one trial of Experiment 2.

(b) Let θ be the probability of success for the procedure.

(i) Find the MLE for θ in Experiment 1.

(ii) Find the MLE for θ in Experiment 2.

Write your answer in terms of \bar{x}

Parts (c-e) use the following table which gives counts from 40 trials, e.g. $x = 2$ occurred in 7 out of 40 trials.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
counts	18	12	7	10	3	2	3	2	1	1	0	1	0

(c) Compute \bar{x} the average number of successes in the 60 trials.

(d) (i) Assuming the data came from Experiment 1, show the MLE for θ is $\hat{\theta} = 0.19167$.

(ii) Assuming the data came from Experiment 2, show the MLE for θ is $\hat{\theta} = 0.69697$.

(e) (i) Do a χ^2 goodness of fit to test the estimate to part d(i). Find the p -value and use a significance of 0.05.

(ii) Repeat part (i) using the estimate in d(ii).

Don't forget to combine the last bins so that all count are at least 5. In this test the number of degrees of freedom is the number of bins - 2.

21. Other good NHST problems are in psets 7 and 8, including two-sample t test, ANOVA, and F-test for equal variance.

Standard normal table of left tail probabilities.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.00	0.0000	-2.00	0.0228	0.00	0.5000	2.00	0.9772
-3.95	0.0000	-1.95	0.0256	0.05	0.5199	2.05	0.9798
-3.90	0.0000	-1.90	0.0287	0.10	0.5398	2.10	0.9821
-3.85	0.0001	-1.85	0.0322	0.15	0.5596	2.15	0.9842
-3.80	0.0001	-1.80	0.0359	0.20	0.5793	2.20	0.9861
-3.75	0.0001	-1.75	0.0401	0.25	0.5987	2.25	0.9878
-3.70	0.0001	-1.70	0.0446	0.30	0.6179	2.30	0.9893
-3.65	0.0001	-1.65	0.0495	0.35	0.6368	2.35	0.9906
-3.60	0.0002	-1.60	0.0548	0.40	0.6554	2.40	0.9918
-3.55	0.0002	-1.55	0.0606	0.45	0.6736	2.45	0.9929
-3.50	0.0002	-1.50	0.0668	0.50	0.6915	2.50	0.9938
-3.45	0.0003	-1.45	0.0735	0.55	0.7088	2.55	0.9946
-3.40	0.0003	-1.40	0.0808	0.60	0.7257	2.60	0.9953
-3.35	0.0004	-1.35	0.0885	0.65	0.7422	2.65	0.9960
-3.30	0.0005	-1.30	0.0968	0.70	0.7580	2.70	0.9965
-3.25	0.0006	-1.25	0.1056	0.75	0.7734	2.75	0.9970
-3.20	0.0007	-1.20	0.1151	0.80	0.7881	2.80	0.9974
-3.15	0.0008	-1.15	0.1251	0.85	0.8023	2.85	0.9978
-3.10	0.0010	-1.10	0.1357	0.90	0.8159	2.90	0.9981
-3.05	0.0011	-1.05	0.1469	0.95	0.8289	2.95	0.9984
-3.00	0.0013	-1.00	0.1587	1.00	0.8413	3.00	0.9987
-2.95	0.0016	-0.95	0.1711	1.05	0.8531	3.05	0.9989
-2.90	0.0019	-0.90	0.1841	1.10	0.8643	3.10	0.9990
-2.85	0.0022	-0.85	0.1977	1.15	0.8749	3.15	0.9992
-2.80	0.0026	-0.80	0.2119	1.20	0.8849	3.20	0.9993
-2.75	0.0030	-0.75	0.2266	1.25	0.8944	3.25	0.9994
-2.70	0.0035	-0.70	0.2420	1.30	0.9032	3.30	0.9995
-2.65	0.0040	-0.65	0.2578	1.35	0.9115	3.35	0.9996
-2.60	0.0047	-0.60	0.2743	1.40	0.9192	3.40	0.9997
-2.55	0.0054	-0.55	0.2912	1.45	0.9265	3.45	0.9997
-2.50	0.0062	-0.50	0.3085	1.50	0.9332	3.50	0.9998
-2.45	0.0071	-0.45	0.3264	1.55	0.9394	3.55	0.9998
-2.40	0.0082	-0.40	0.3446	1.60	0.9452	3.60	0.9998
-2.35	0.0094	-0.35	0.3632	1.65	0.9505	3.65	0.9999
-2.30	0.0107	-0.30	0.3821	1.70	0.9554	3.70	0.9999
-2.25	0.0122	-0.25	0.4013	1.75	0.9599	3.75	0.9999
-2.20	0.0139	-0.20	0.4207	1.80	0.9641	3.80	0.9999
-2.15	0.0158	-0.15	0.4404	1.85	0.9678	3.85	0.9999
-2.10	0.0179	-0.10	0.4602	1.90	0.9713	3.90	1.0000
-2.05	0.0202	-0.05	0.4801	1.95	0.9744	3.95	1.0000

$\Phi(z) = P(Z \leq z)$ for $N(0, 1)$.

(Use interpolation to estimate z values to a 3rd decimal place.)

t-table of left tail probabilities. (The tables show $P(T < t)$ for $T \sim t(df)$.)

t\df	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5628	0.5700	0.5729	0.5744	0.5753	0.5760	0.5764	0.5768	0.5770
0.4	0.6211	0.6361	0.6420	0.6452	0.6472	0.6485	0.6495	0.6502	0.6508
0.6	0.6720	0.6953	0.7046	0.7096	0.7127	0.7148	0.7163	0.7174	0.7183
0.8	0.7148	0.7462	0.7589	0.7657	0.7700	0.7729	0.7750	0.7766	0.7778
1.0	0.7500	0.7887	0.8045	0.8130	0.8184	0.8220	0.8247	0.8267	0.8283
1.2	0.7789	0.8235	0.8419	0.8518	0.8581	0.8623	0.8654	0.8678	0.8696
1.4	0.8026	0.8518	0.8720	0.8829	0.8898	0.8945	0.8979	0.9005	0.9025
1.6	0.8222	0.8746	0.8960	0.9076	0.9148	0.9196	0.9232	0.9259	0.9280
1.8	0.8386	0.8932	0.9152	0.9269	0.9341	0.9390	0.9426	0.9452	0.9473
2.0	0.8524	0.9082	0.9303	0.9419	0.9490	0.9538	0.9572	0.9597	0.9617
2.2	0.8642	0.9206	0.9424	0.9537	0.9605	0.9649	0.9681	0.9705	0.9723
2.4	0.8743	0.9308	0.9521	0.9628	0.9692	0.9734	0.9763	0.9784	0.9801
2.6	0.8831	0.9392	0.9598	0.9700	0.9759	0.9797	0.9823	0.9842	0.9856
2.8	0.8908	0.9463	0.9661	0.9756	0.9810	0.9844	0.9867	0.9884	0.9896
3.0	0.8976	0.9523	0.9712	0.9800	0.9850	0.9880	0.9900	0.9915	0.9925
3.2	0.9036	0.9573	0.9753	0.9835	0.9880	0.9907	0.9925	0.9937	0.9946
3.4	0.9089	0.9617	0.9788	0.9864	0.9904	0.9928	0.9943	0.9953	0.9961
3.6	0.9138	0.9654	0.9816	0.9886	0.9922	0.9943	0.9956	0.9965	0.9971
3.8	0.9181	0.9686	0.9840	0.9904	0.9937	0.9955	0.9966	0.9974	0.9979
4.0	0.9220	0.9714	0.9860	0.9919	0.9948	0.9964	0.9974	0.9980	0.9984

t\df	10	11	12	13	14	15	16	17	18	19
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5773	0.5774	0.5776	0.5777	0.5778	0.5779	0.5780	0.5781	0.5781	0.5782
0.4	0.6512	0.6516	0.6519	0.6522	0.6524	0.6526	0.6528	0.6529	0.6531	0.6532
0.6	0.7191	0.7197	0.7202	0.7206	0.7210	0.7213	0.7215	0.7218	0.7220	0.7222
0.8	0.7788	0.7797	0.7804	0.7810	0.7815	0.7819	0.7823	0.7826	0.7829	0.7832
1.0	0.8296	0.8306	0.8315	0.8322	0.8329	0.8334	0.8339	0.8343	0.8347	0.8351
1.2	0.8711	0.8723	0.8734	0.8742	0.8750	0.8756	0.8762	0.8767	0.8772	0.8776
1.4	0.9041	0.9055	0.9066	0.9075	0.9084	0.9091	0.9097	0.9103	0.9107	0.9112
1.6	0.9297	0.9310	0.9322	0.9332	0.9340	0.9348	0.9354	0.9360	0.9365	0.9370
1.8	0.9490	0.9503	0.9515	0.9525	0.9533	0.9540	0.9546	0.9552	0.9557	0.9561
2.0	0.9633	0.9646	0.9657	0.9666	0.9674	0.9680	0.9686	0.9691	0.9696	0.9700
2.2	0.9738	0.9750	0.9759	0.9768	0.9774	0.9781	0.9786	0.9790	0.9794	0.9798
2.4	0.9813	0.9824	0.9832	0.9840	0.9846	0.9851	0.9855	0.9859	0.9863	0.9866
2.6	0.9868	0.9877	0.9884	0.9890	0.9895	0.9900	0.9903	0.9907	0.9910	0.9912
2.8	0.9906	0.9914	0.9920	0.9925	0.9929	0.9933	0.9936	0.9938	0.9941	0.9943
3.0	0.9933	0.9940	0.9945	0.9949	0.9952	0.9955	0.9958	0.9960	0.9962	0.9963

t\df	20	21	22	23	24	25	26	27	28	29
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5782	0.5783	0.5783	0.5784	0.5784	0.5785	0.5785	0.5785	0.5785	0.5786
0.4	0.6533	0.6534	0.6535	0.6536	0.6537	0.6537	0.6538	0.6538	0.6539	0.6540
0.6	0.7224	0.7225	0.7227	0.7228	0.7229	0.7230	0.7231	0.7232	0.7233	0.7234
0.8	0.7834	0.7837	0.7839	0.7841	0.7842	0.7844	0.7845	0.7847	0.7848	0.7849
1.0	0.8354	0.8357	0.8359	0.8361	0.8364	0.8366	0.8367	0.8369	0.8371	0.8372
1.2	0.8779	0.8782	0.8785	0.8788	0.8791	0.8793	0.8795	0.8797	0.8799	0.8801
1.4	0.9116	0.9119	0.9123	0.9126	0.9128	0.9131	0.9133	0.9136	0.9138	0.9139
1.6	0.9374	0.9377	0.9381	0.9384	0.9387	0.9389	0.9392	0.9394	0.9396	0.9398
1.8	0.9565	0.9569	0.9572	0.9575	0.9578	0.9580	0.9583	0.9585	0.9587	0.9589
2.0	0.9704	0.9707	0.9710	0.9713	0.9715	0.9718	0.9720	0.9722	0.9724	0.9725
2.2	0.9801	0.9804	0.9807	0.9809	0.9812	0.9814	0.9816	0.9817	0.9819	0.9820
2.4	0.9869	0.9871	0.9874	0.9876	0.9877	0.9879	0.9881	0.9882	0.9884	0.9885
2.6	0.9914	0.9916	0.9918	0.9920	0.9921	0.9923	0.9924	0.9925	0.9926	0.9927
2.8	0.9945	0.9946	0.9948	0.9949	0.9950	0.9951	0.9952	0.9953	0.9954	0.9955
3.0	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9971	0.9972	0.9973

t\df	30	31	32	33	34	35	36	37	38	39
0.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.2	0.5786	0.5786	0.5786	0.5786	0.5787	0.5787	0.5787	0.5787	0.5787	0.5787
0.4	0.6540	0.6541	0.6541	0.6541	0.6542	0.6542	0.6542	0.6543	0.6543	0.6543
0.6	0.7235	0.7236	0.7236	0.7237	0.7238	0.7238	0.7239	0.7239	0.7240	0.7240
0.8	0.7850	0.7851	0.7852	0.7853	0.7854	0.7854	0.7855	0.7856	0.7857	0.7857
1.0	0.8373	0.8375	0.8376	0.8377	0.8378	0.8379	0.8380	0.8381	0.8382	0.8383
1.2	0.8802	0.8804	0.8805	0.8807	0.8808	0.8809	0.8810	0.8811	0.8812	0.8813
1.4	0.9141	0.9143	0.9144	0.9146	0.9147	0.9148	0.9150	0.9151	0.9152	0.9153
1.6	0.9400	0.9401	0.9403	0.9404	0.9406	0.9407	0.9408	0.9409	0.9411	0.9412
1.8	0.9590	0.9592	0.9594	0.9595	0.9596	0.9598	0.9599	0.9600	0.9601	0.9602
2.0	0.9727	0.9728	0.9730	0.9731	0.9732	0.9733	0.9735	0.9736	0.9737	0.9738
2.2	0.9822	0.9823	0.9824	0.9825	0.9826	0.9827	0.9828	0.9829	0.9830	0.9831
2.4	0.9886	0.9887	0.9888	0.9889	0.9890	0.9891	0.9892	0.9892	0.9893	0.9894
2.6	0.9928	0.9929	0.9930	0.9931	0.9932	0.9932	0.9933	0.9933	0.9934	0.9935
2.8	0.9956	0.9956	0.9957	0.9958	0.9958	0.9959	0.9959	0.9960	0.9960	0.9960
3.0	0.9973	0.9974	0.9974	0.9974	0.9975	0.9975	0.9976	0.9976	0.9976	0.9977

χ^2 -table of right tail critical values

The table shows $c_{df,p}$ = the $1 - p$ quantile of $\chi^2(df)$.

In R notation $c_{df,p} = \text{qchisq}(1-p, m)$.

df \ p	0.010	0.025	0.050	0.100	0.200	0.300	0.500	0.700	0.800	0.900	0.950	0.975	0.990
1	6.63	5.02	3.84	2.71	1.64	1.07	0.45	0.15	0.06	0.02	0.00	0.00	0.00
2	9.21	7.38	5.99	4.61	3.22	2.41	1.39	0.71	0.45	0.21	0.10	0.05	0.02
3	11.34	9.35	7.81	6.25	4.64	3.66	2.37	1.42	1.01	0.58	0.35	0.22	0.11
4	13.28	11.14	9.49	7.78	5.99	4.88	3.36	2.19	1.65	1.06	0.71	0.48	0.30
5	15.09	12.83	11.07	9.24	7.29	6.06	4.35	3.00	2.34	1.61	1.15	0.83	0.55
6	16.81	14.45	12.59	10.64	8.56	7.23	5.35	3.83	3.07	2.20	1.64	1.24	0.87
7	18.48	16.01	14.07	12.02	9.80	8.38	6.35	4.67	3.82	2.83	2.17	1.69	1.24
8	20.09	17.53	15.51	13.36	11.03	9.52	7.34	5.53	4.59	3.49	2.73	2.18	1.65
9	21.67	19.02	16.92	14.68	12.24	10.66	8.34	6.39	5.38	4.17	3.33	2.70	2.09
10	23.21	20.48	18.31	15.99	13.44	11.78	9.34	7.27	6.18	4.87	3.94	3.25	2.56
16	32.00	28.85	26.30	23.54	20.47	18.42	15.34	12.62	11.15	9.31	7.96	6.91	5.81
17	33.41	30.19	27.59	24.77	21.61	19.51	16.34	13.53	12.00	10.09	8.67	7.56	6.41
18	34.81	31.53	28.87	25.99	22.76	20.60	17.34	14.44	12.86	10.86	9.39	8.23	7.01
19	36.19	32.85	30.14	27.20	23.90	21.69	18.34	15.35	13.72	11.65	10.12	8.91	7.63
20	37.57	34.17	31.41	28.41	25.04	22.77	19.34	16.27	14.58	12.44	10.85	9.59	8.26
21	38.93	35.48	32.67	29.62	26.17	23.86	20.34	17.18	15.44	13.24	11.59	10.28	8.90
22	40.29	36.78	33.92	30.81	27.30	24.94	21.34	18.10	16.31	14.04	12.34	10.98	9.54
23	41.64	38.08	35.17	32.01	28.43	26.02	22.34	19.02	17.19	14.85	13.09	11.69	10.20
24	42.98	39.36	36.42	33.20	29.55	27.10	23.34	19.94	18.06	15.66	13.85	12.40	10.86
25	44.31	40.65	37.65	34.38	30.68	28.17	24.34	20.87	18.94	16.47	14.61	13.12	11.52
30	50.89	46.98	43.77	40.26	36.25	33.53	29.34	25.51	23.36	20.60	18.49	16.79	14.95
31	52.19	48.23	44.99	41.42	37.36	34.60	30.34	26.44	24.26	21.43	19.28	17.54	15.66
32	53.49	49.48	46.19	42.58	38.47	35.66	31.34	27.37	25.15	22.27	20.07	18.29	16.36
33	54.78	50.73	47.40	43.75	39.57	36.73	32.34	28.31	26.04	23.11	20.87	19.05	17.07
34	56.06	51.97	48.60	44.90	40.68	37.80	33.34	29.24	26.94	23.95	21.66	19.81	17.79
35	57.34	53.20	49.80	46.06	41.78	38.86	34.34	30.18	27.84	24.80	22.47	20.57	18.51
40	63.69	59.34	55.76	51.81	47.27	44.16	39.34	34.87	32.34	29.05	26.51	24.43	22.16
41	64.95	60.56	56.94	52.95	48.36	45.22	40.34	35.81	33.25	29.91	27.33	25.21	22.91
42	66.21	61.78	58.12	54.09	49.46	46.28	41.34	36.75	34.16	30.77	28.14	26.00	23.65
43	67.46	62.99	59.30	55.23	50.55	47.34	42.34	37.70	35.07	31.63	28.96	26.79	24.40
44	68.71	64.20	60.48	56.37	51.64	48.40	43.34	38.64	35.97	32.49	29.79	27.57	25.15
45	69.96	65.41	61.66	57.51	52.73	49.45	44.34	39.58	36.88	33.35	30.61	28.37	25.90
46	71.20	66.62	62.83	58.64	53.82	50.51	45.34	40.53	37.80	34.22	31.44	29.16	26.66
47	72.44	67.82	64.00	59.77	54.91	51.56	46.34	41.47	38.71	35.08	32.27	29.96	27.42
48	73.68	69.02	65.17	60.91	55.99	52.62	47.34	42.42	39.62	35.95	33.10	30.75	28.18
49	74.92	70.22	66.34	62.04	57.08	53.67	48.33	43.37	40.53	36.82	33.93	31.55	28.94

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