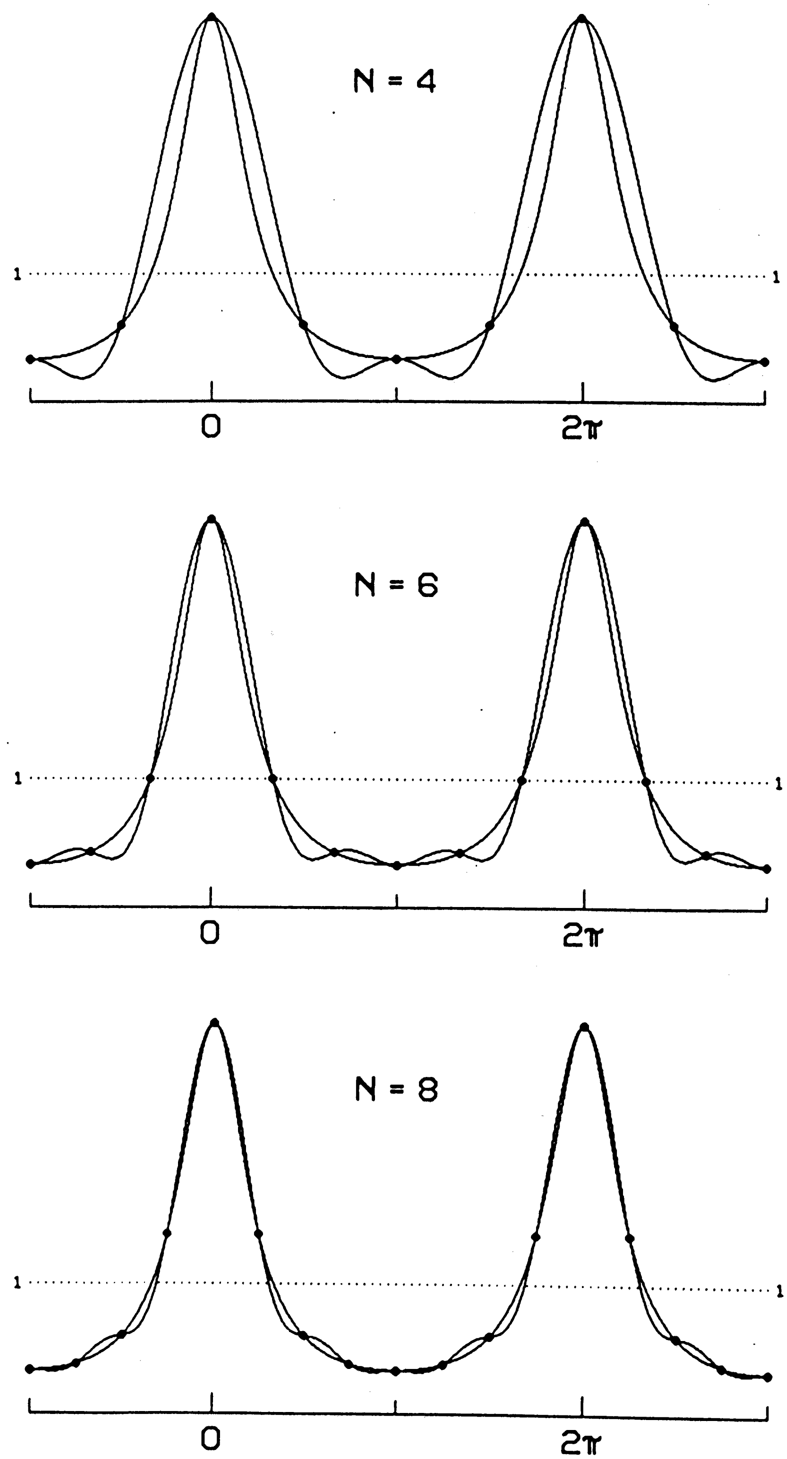
18.04 Handout Mon 08 Dec 03

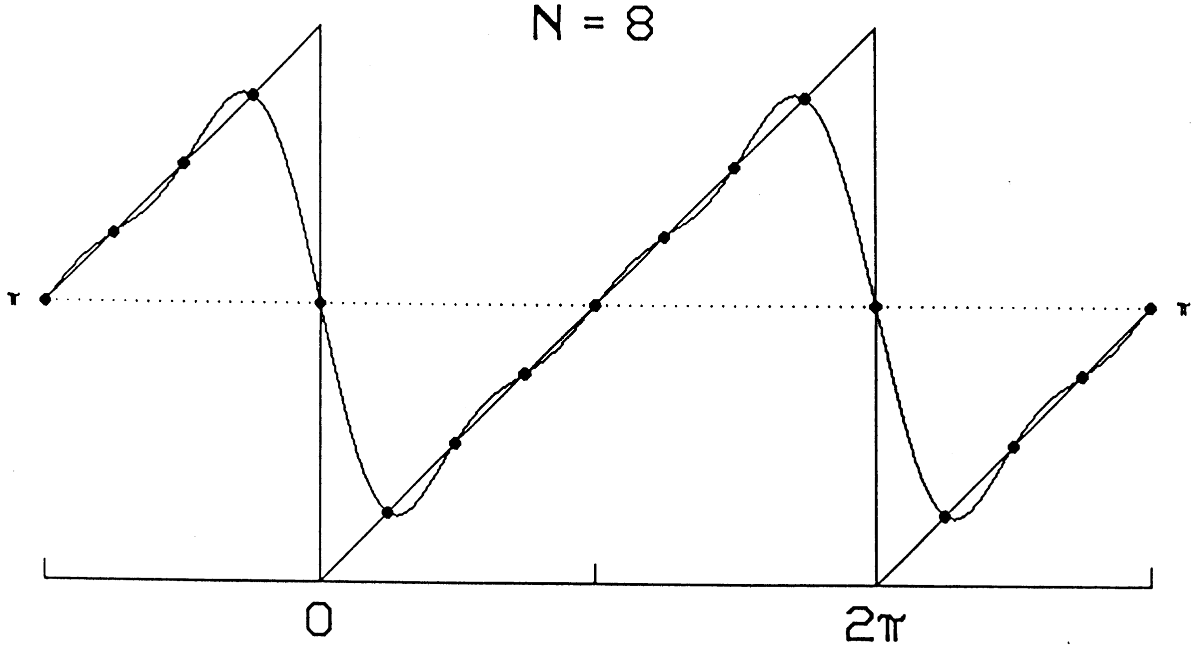
on **FOURIER INTERPOLATIONS**

toward $f(x) = 3 / (5 - 4 \cos x)$ and **FFT**

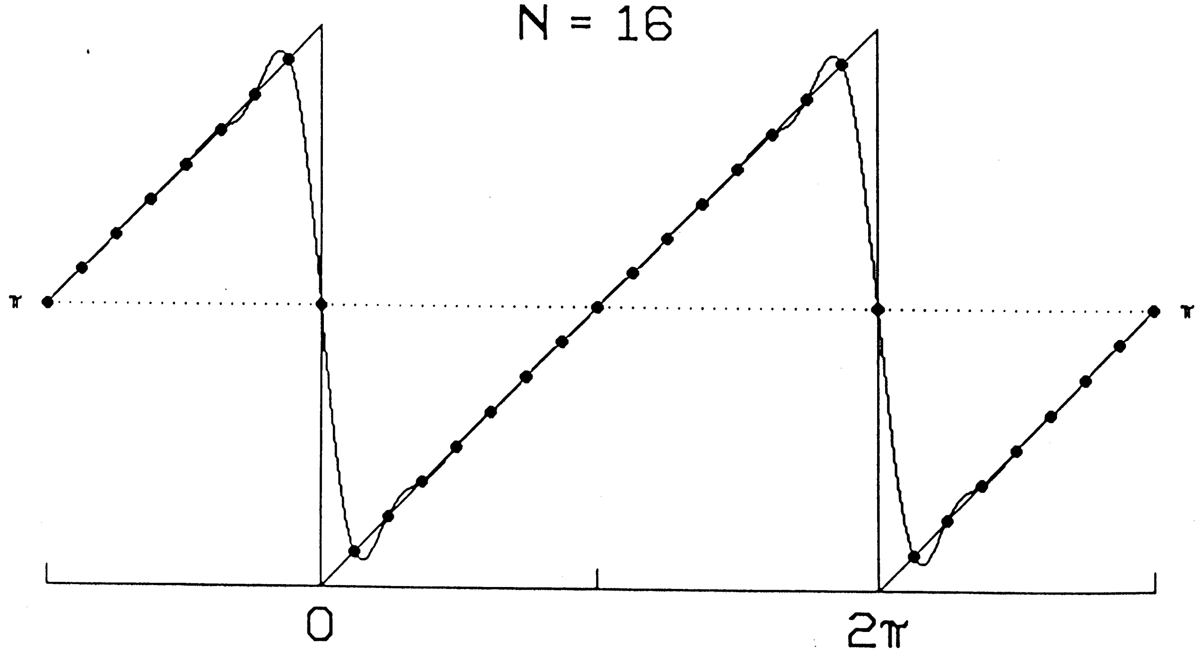


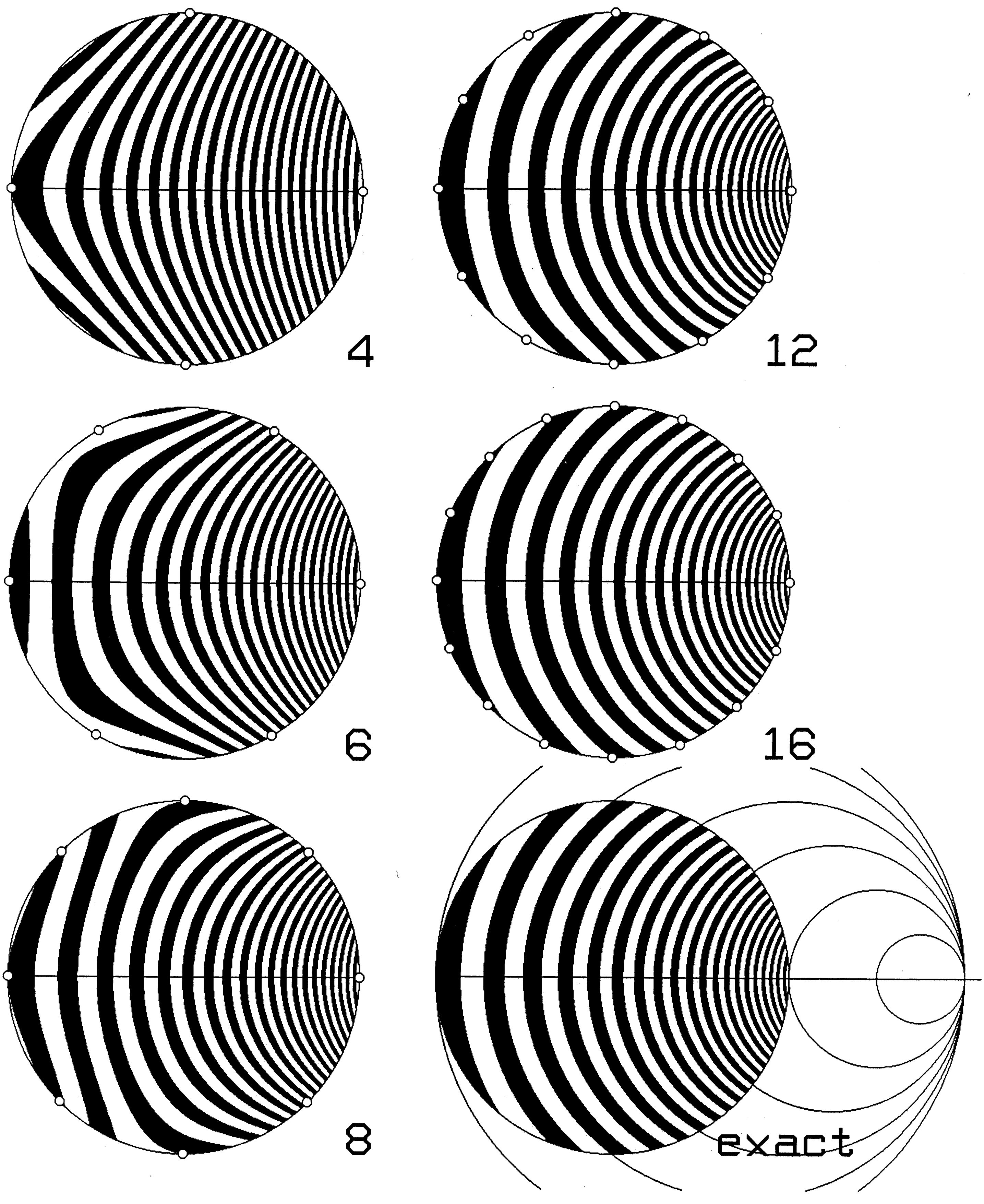
FFT at Work ... on the triangle wave trw(x)

N = 8



N = 16




4
12
6
16
8
exact

Here the approximate solutions (from Fourier **interpolations** along $r = 1$) are

$$T_4(r,\theta) = \frac{17}{15} + 2\times\frac{10}{15}\, r\cos\theta + \frac{8}{15}\, r^2\cos 2\theta$$

$$T_6(r,\theta) = \frac{65}{63} + 2\times\frac{34}{63}\, r\cos\theta + 2\times\frac{20}{63}\, r^2\cos 2\theta + \frac{16}{63}\, r^3\cos 3\theta$$

$$T_8(r,\theta) = \frac{257}{255} + 2\times\frac{130}{255}\, r\cos\theta + 2\times\frac{68}{255}\, r^2\cos 2\theta + 2\times\frac{40}{255}\, r^3\cos 3\theta + \frac{32}{255}\, r^4\cos 4\theta$$

et cetera, whereas the exact solution is

$$T(r,\theta) = \frac{4 - r^2}{4 - 4r\cos\theta + r^2} = \mathrm{Re}\left(\frac{2+z}{2-z}\right) .$$

That the former approach the latter quite rapidly can be seen not only from the "zebra" plots in the front — with convenient spacings $\Delta T = 1/15$ — but also from specific comparisons like

$$T_4(\tfrac{1}{2},0) = \frac{5}{3} + \frac{4}{15}$$
$$T_6(\tfrac{1}{2},0) = \frac{5}{3} + \frac{6}{63}$$
$$T_8(\tfrac{1}{2},0) = \frac{5}{3} + \frac{8}{255}$$

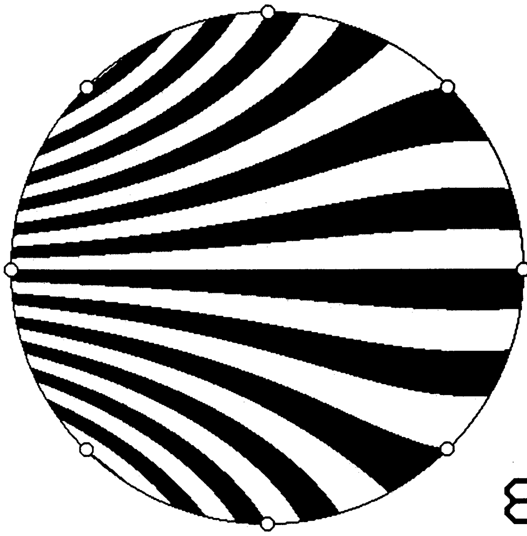
and

$$T_{12}(\tfrac{1}{2},0) = \frac{5}{3} + \frac{12}{4095}$$
$$T_{16}(\tfrac{1}{2},0) = \frac{5}{3} + \frac{16}{65535}$$

versus the precise answer

$$T(\tfrac{1}{2},0) = \frac{5}{3} .$$

18.04 Triangle wave ... revisited Mon 08 Dec 03



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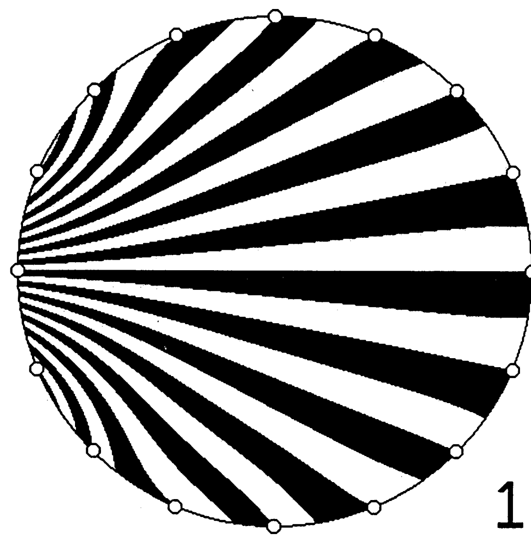

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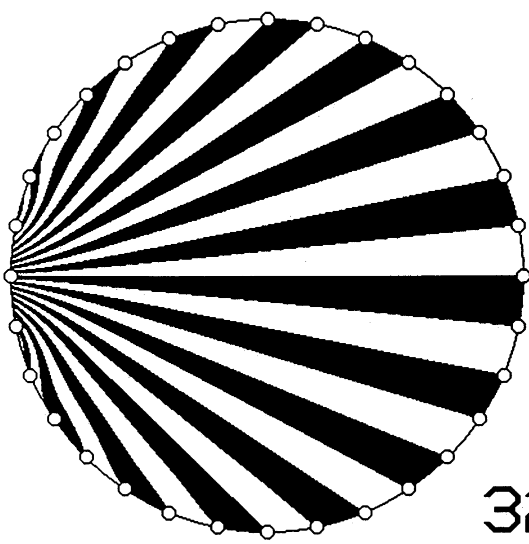

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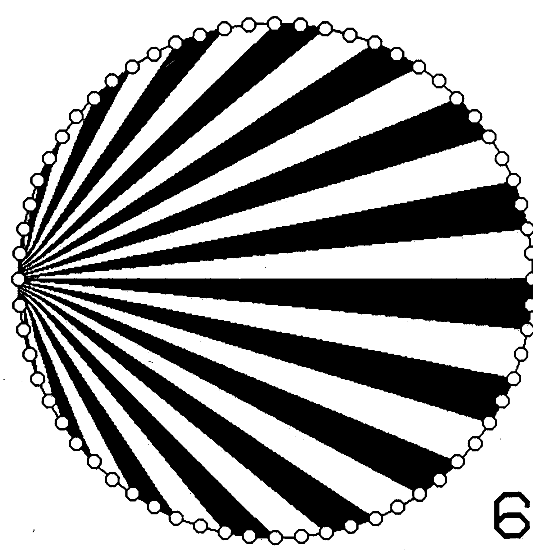

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