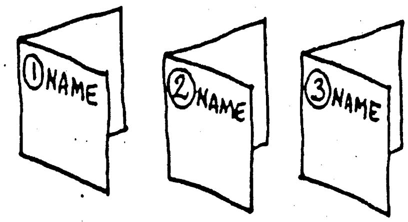
18.04 Exam #2 Friday, October 31, 2003

CLOSED BOOK . . . and **<u>NO calculators</u>**

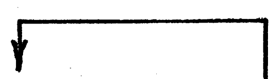
As in the past, please . . .



(1) Let the <u>non-constant</u> function $f(z)$ be analytic within and on the circle $|z-1| = 2$, and also suppose that $|f(z)| = 3$ everywhere on that circular boundary. Then use the Maximum Modulus Principle — state it clearly but do not bother to prove it — to show that this $f(z)$ must have at least one zero within that circle.

HINT: Think of $g(z) = 1/f$.

(2) Use residue calculus to evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{2 + x^2}{4 + x^4}\, dx$.

(3) Use residue calculus and a path shaped like  to evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{\cosh^3 x} = 8\int_{-\infty}^{\infty} \frac{dx}{(e^x + e^{-x})^3} .$$