Exam Number 3 for 18.04, MIT (Fall 1999).

Due on the last day of classes, Fall 1999.

Rodolfo R. Rosales* Boris Schlittgen† Zhaohui Zhang‡

December 2, 1999.

Contents

*MIT, Department of Mathematics, room 2-337, Cambridge, MA 02139.

†MIT, Department of Mathematics, room 2-490, Cambridge, MA 02139.

‡MIT, Department of Mathematics, room 2-229, Cambridge, MA 02139.

1 Instructions.

1. **Penalties may/will result for failure to follow the rules below.**
2. **Do it alone.** You **can consult only** with the lecturer and/or the recitation instructors.
3. You can use the textbook, your class notes, the hand outs and the problem set answers. **Nothing else.** Be **specific** with the references (e.g.: "Using Theorem 87, p. 986 in the book") and **explain** how the reference fits into the answer. You **cannot** use the answers to the problems at the end of the book (except for those assigned, where a solution was provided with the problem set answers).
4. There is no time limit, but a few hours should be enough.
5. **Write the answers to each problem on separate pages, and** *PRINT* **your name and problem being solved at the head of each page** (e.g.: *18.04, J. Doe, Exam #3, Problem #77, page 2 of 29.*
6. **Staple** the whole exam.
7. In all your solutions show your reasoning, explaining carefully what you are doing. **Use English, not just mathematical symbols.** You play dice with unjustified steps. Please: no "chicken scratches" or arrows on the side of the page leading from one piece of an argument to another and so on. *If a particular thing is illegible,* **write it again.**

   The answers MUST be readable; this means (in particular) that they have to be written in a large enough font and with a dark enough pen/pencil!

   Put a box around your **final answers** (or use any other device you want, but make it so they **can be found easily).**

   Answers that do not satisfy these criteria will be ignored.
8. Start **early.** Do not wait till the night before it is due.
9. The problems are actually neither hard nor do they require much cumbersome calculation, **if** you think about them carefully and know what you are doing and where you are going. If you find yourself in the middle of a long messy answer, **stop** and think again about your strategy. It may not be a good way (maybe not even a way) to solve the problem.

   Warning: Present a solution **only** if you have some reasonably good idea of what is going on in a problem. Do not just write something in the hope of getting some partial credit; partial credit will be given when it is due, but **NEGATIVE CREDIT** will accrue for any gross error or similar (i.e.: beware of writing nonsense!)

2 Problems.

2.1 Problem 1999.3.1 (10 points).

Calculate the residue (at $z = 0$) of $g(z) = \frac{d}{dz} f(z)$, where: $f(z) = \frac{1}{z^3} e^{\cos(z)} \cos(z) + \frac{1}{\text{Log}(1+z)}$.

2.2 Problem 1999.3.2 (10 points).

Calculate the integral: $I = \int_{-\infty}^{\infty} \frac{dx}{(1+x^2)\sin(x-i)}$.

2.3 Problem 1999.3.3 (10 points).

Calculate the Laurent expansions, valid on: **(a)** $\sqrt{2} < |z| < 2$,

(b) $\alpha < |z - i\sqrt{2}| < 2\sqrt{2}$ **(what is α?)**,

for $f(z) = \frac{1}{2+z^2} + \frac{1}{2-z}$.

2.4 Problem 1999.3.4 (15 points).

Calculate, for any $k \neq 0$ real, the integral: $I = I(k) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{1+x^2} x\, dx$.

2.5 Problem 1999.3.5 (15 points).

Calculate (for any $-1 < \alpha < \beta - 1$) the integral: $I = I(\alpha, \beta) = \int_0^{\infty} \frac{x^\alpha}{1+x^\beta} dx$.
Here $x^\alpha > 0$ and $x^\beta > 0$ for any $x > 0$.

2.6 Problem 1999.3.6 (10 points).

Let $f(z) = \sqrt{1 - \sin(z)}$. **(a)** Find and classify the singularities and zeros of this function.

(b) Is this function entire? If so, show it; if not, explain why not.

2.7 Problem 1999.3.7 (15 points).

Show that all the roots of the equation $z^6 - 5z^2 + 10 = 0$ lie in the annulus $1 < |z| < 2$.

2.8 Problem 1999.3.8 (15 points).

Find the first five terms in the Laurent expansion for $\coth(z)$ valid on $0 < |z| < \alpha$. **What is α?**

THE END.