Summary

In summary, the procedure of sketching trajectories of the 2×2 linear homogeneous system $\mathbf{x}' = A\mathbf{x}$, where A is a constant matrix, is the following.

Begin by finding the eigenvalues of A.

1. If they are real, distinct, and non-zero:

   a) find the corresponding eigenvectors;

   b) draw in the corresponding solutions whose trajectories are rays. Use the sign of the eigenvalue to determine the direction of motion as t increases; indicate it with an arrowhead on the ray;

   c) draw in some nearby smooth curves, with arrowheads indicating the direction of motion:
      (i) if the eigenvalues have opposite signs, this is easy;
      (ii) if the eigenvalues have the same sign, determine which is the dominant term in the solution for $t \gg 1$ and $t \ll -1$, and use this to determine which rays the trajectories are tangent to, near the origin, and which rays they are parallel to, away from the origin. (Or use the node-sketching principle.)

2. If the eigenvalues are complex, $a \pm bi$, the trajectories will be

   a) ellipses if $a = 0$

   b) spirals if $a \neq 0$; inward if $a < 0$, outward if $a > 0$.

   In all cases, determine the direction of motion by using the system $\mathbf{x}' = A\mathbf{x}$ to find one velocity vector.

3. The details in the other cases (eigenvalues repeated, or zero) will be left as exercises using the reasoning in this note.

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