

18.03SC Practice Problems 34

Trace-Determinant Plane

Solution suggestions

1. *On the trace-determinant plane, where can you guarantee that any matrix with that value of trace and determinant is stable? Unstable? Neutrally stable? Are there there any values of the trace and determinant for which there are matrices exhibiting more than one type of limiting behavior?*

Stable solutions occur if and only if both eigenvalues have negative real part. This happens exactly when the trace is negative and the determinant is positive - so, at all points inside the second quadrant in the (T, D) plane.

Unstable solutions occur when there is at least one eigenvalue with positive real part. This happens either when the determinant is negative, or when the trace is positive - so, at all points inside the first, third, and fourth quadrants on the (T, D) plane.

Neutrally stable solutions occur either when the trace is zero and the determinant is positive (giving periodic solutions), or when the determinant is zero and the trace is negative (giving combs).

When the trace and determinant are both zero, the zero matrix gives neutrally stable behavior, while the matrix that has exactly one nonzero entry, in the upper right, gives unstable behavior.

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18.03SC Differential Equations
Fall 2011

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