

Pset 10 Part I

For the following systems, the origin is clearly a critical point. Give its geometric type and stability, and sketch some nearby trajectories of the system.

Problem 1:

$$\begin{aligned}x' &= x - y + xy \\y' &= 3x - 2y - xy\end{aligned}$$

Problem 2:

$$\begin{aligned}x' &= x + 2x^2 - y^2 \\y' &= x - 2y + x^3\end{aligned}$$

For the following systems carry out the linearization for sketching trajectories. Find the critical points, analyze each, draw in nearby trajectories, then add some other trajectories compatible with the ones you have drawn; when necessary, sketch in a well-chosen vector from the vector field to help.

Problem 3:

$$\begin{aligned}x' &= 1 - y \\y' &= x^2 - y^2\end{aligned}$$

Problem 4:

$$\begin{aligned}x' &= x - x^2 - xy \\y' &= 3y - xy - 2y^2\end{aligned}$$

Problem 5: Structural stability:

The following system has a critical point at the origin:

$$x' = 3x - y + x^2 + y^2, \quad y' = -6x + 2y + 3xy.$$

For that critical point, find the geometric type and stability of the corresponding linearized system, and then tell what the possibilities would be for the corresponding critical point of the given non-linear system.

Problem 6: Structural stability:

The following system has one critical point whose linearization is not structurally stable:

$$x' = y, \quad y' = x(1 - x).$$

Begin by finding the critical points and determining the type of the corresponding linearized system at each of the critical points. Then in each case, sketch several pictures showing the different ways the trajectories of the non-linear system *might* look.

For the next three problems, find and classify the critical points of the given non-linear system. Then use this information to give a rough sketch of the solution curves.

Problem 7:

$$\begin{aligned}x' &= 60x - 4x^2 - 3xy \\y' &= 42y - 2y^2 - 3xy\end{aligned}$$

Problem 8:

$$\begin{aligned}x' &= 5x - x^2 - xy \\y' &= -2y + xy\end{aligned}$$

Problem 9:

$$\begin{aligned}x' &= x^2 - 2x - xy \\y' &= y^2 - 4y + xy\end{aligned}$$

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