

Pset 10 Part II

Problem 1: For the 2×2 autonomous system

$$\begin{aligned}x' &= x - 2y + \frac{1}{4}x^2 \\y' &= 5x - y - y^2\end{aligned}$$

(a) Find the critical points.

Note: you will get a quartic polynomial; to help you solve it we'll tell you that one root is 0 and another is a positive integer no larger than 5. There are only two critical points, but you'll need to find the other two roots of this polynomial and show they don't give critical points.

(b) Find the linearized system at each critical point. Then carry out the procedure described in the session on Linearization, culminating in one sketch which includes the trajectories near each of the critical points, and a *guess* at how they all fit together.

Problem 2: Classify as structurally stable or not stable each of the critical points found in the previous problem. Based on this what can you say for sure about the behavior of the trajectories near each critical point? How does this relate to your hand 'guess-sketch'?

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