

## Companion Matrices

Open up the applet *Linear phase portrait: Matrix entry* again and uncheck *companion matrix*. Let's see what happens to Romeo and Juliet. Enter the matrix corresponding to their system:

$$A = \begin{pmatrix} 0.25 & 1 \\ -1.125 & 0.75 \end{pmatrix}.$$

What do the trajectories look like? How can they be interpreted?

Remember that  $x$  corresponds to  $R$ , and  $y$  to  $J$ . Let us start at  $(.5,0)$ : Romeo is fond of Juliet, but she is neutral towards him. However, she does notice that he is fond of her and this makes her somewhat hostile. As she becomes more distant, his affection wanes. Eventually, he is neutral and she really doesn't like him: the trajectory is (roughly) at  $(0,-1)$ . This continues; presently he stays away from her, and this very fact makes her more interested. She warms to him; he notices, and  $R'$ , while still negative, start to increase. Eventually she is neutral and the trajectory crosses the  $x$ -axis again, around  $(-2.4,0)$ . He then starts to feel better towards her, but still stays away; now both his attitude and hers cause her to feel progressively more well disposed towards him. This causes him to continue to warm to her. Following this around, you wind up at  $J = 0$  again, but now  $R$  has increased: it is already outside the applet's screen. This is a cyclical evolution, but with each cycle the intensity of feelings increases. We all know the sad outcome.

Now check *companion matrix* again. This corresponds to doing elimination followed by anti-elimination, as in example 2 in the note on the companion matrix. What do the trajectories look like? This should confirm the answer that we obtained analytically for example 1 in section text: companion matrix. How did the trajectories change when you checked 'companion matrix'? Note that  $x$  is still  $R$ , but  $y$  is now  $R'$ , not  $J$ . The fact that the pictures did not change dramatically is no accident. We will learn why in the phase portraits session.

Play around with the applet a bit more, entering some other systems and then clicking *companion matrix* (thus performing elimination followed by anti-elimination). What sorts of things do you notice?

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