

Part II Problems and Solutions

Problem 1: [Second order ODEs via Laplace transform] Find the unit impulse response of the following operators by means of the Laplace transform.

(a) $3D^2 + 6D + 6I$.

(b) $D^4 - I$.

Solution: (a) $w(t)$ has Laplace transform $W(s) = \frac{1}{3s^2 + 6s + 6} = \frac{1}{3} \frac{1}{(s+1)^2 + 1}$. $\mathcal{L}(\sin t) = \frac{1}{s^2+1}$, so by s -shift $w(t) = \frac{1}{3}u(t)e^{-t} \sin t$.

(b) $W(s) = \frac{1}{s^4 - 1}$. The roots of $s^4 - 1$ are ± 1 and $\pm i$, so we can write

$$\frac{1}{s^4 - 1} = \frac{a}{s-1} + \frac{b}{s+1} + \frac{c}{s-i} + \frac{d}{s+i}.$$

Cover-up gives easily $a = b = \frac{1}{4}$, $c = \frac{i}{4}$, $d = -\frac{i}{4}$. So

$$w(t) = u(t) \frac{1}{4} (e^t + e^{-t} + ie^{it} - ie^{-it}) = u(t) \frac{1}{2} (\sinh(t) - \sin(t))$$

(where $\sinh(t) = \frac{1}{2} (e^t + e^{-t})$, the hyperbolic sine function)

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